On the Syntactic Reality of Pragmatic Operators: the Case of Non-Redundancy*

Keny Chatain[a], Philippe Schlenker[a,b]

Abstract. Considerable progress was made at the semantics/pragmatics interface by postulating covert operators that enrich the meaning of some constituents as part of compositional semantics. Three cases in point are Chierchia, Fox and Spector's exhaustivity operator O, used to compute local implicatures; Bochvar's assertion operator A, used to compute local accommodation of presuppositions; and more recently, Blumberg and Goldstein's non-redundancy operator R, used to capture cases of intrusion of non-redundancy constraints in truth conditions. But are these operators syntactically real? In earlier work, we argued that A isn't, and that O must have such a powerful semantics that one can, in near-equivalent fashion, insert it everywhere in the syntax, or encode its effects, operator-free, as part of the semantic procedure. Here we consider the non-redundancy operator R, which purports to derive three types of inferences in the scope of operators: diversity inferences, ignorance inferences, and free choice inferences. Using ellipsis-based tests, we argue that R isn't syntactically real, and we derive its desirable effects from different operator-free mechanisms. First, diversity inferences are lexically triggered presuppositions and not the by-product of a general non-redundancy mechanism. Second, ignorance inferences reflect *pragmatic* non-triviality conditions, not the effect of an operator. Third, free choice inferences can be derived without R by relying on existing and independently motivated theories (a point we only discuss in an appendix).

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[a] Institut Jean-Nicod (ENS - EHESS - CNRS), Département d'Etudes Cognitives, Ecole Normale Supérieure, Paris, France; PSL Research University.
[b] New York University, New York.

1 Debates about the Syntactic Reality of Pragmatic Operators: the case of A and O

In the last 25 years, considerable progress was made in research on the semantics/pragmatics interface by postulating covert operators that enrich the meaning of some constituents as part of compositional semantics. Three cases in point are Bochvar's (1939) assertation operator A, used to compute local accommodation of presuppositions (e.g. Beaver 2001); Chierchia, Fox and Spector's (2012) exhaustivity operator O (also called Exh), used to compute local implicatures; and more recently, Blumberg and Goldstein's (2021a,b) non-redundancy operator R, which was posited to capture cases of intrusion of non-redundancy constraints in truth conditions. But are these operators syntactically real? After summarizing recent doubts pertaining to A and O, we will focus on R and argue that it is not syntactically real, and that its desirable properties are better analyzed by way of (different) operator-free pragmatic conditions.

1.1 Operator-based and operator-free analyses

Recent research has offered a nuanced assessment about the syntactic reality of pragmatic operators. First, when it comes to A, ellipsis-tests suggest that it is not syntactically real, or that it has such a flexible (environment-sensitive) semantics that it can be inserted systematically. Second, ellipsis-tests with O have been inconclusive because the complexity of the crucial examples made judgments difficult. But other tests, based on anaphora, led to the conclusion that the semantics of O is so flexible (environment-sensitive) that, like A, it could be inserted systematically.

If a covert operator is inserted systematically, one can also obtain its semantic effect, operator-free, as part of the interpretive procedure. To take a fictional example, suppose that exhaustification systematically took place, so that *I'll invite Ann or Bill* were always interpreted as *I'll only invite Ann OR Bill*, including upon embedding (this is fanciful, as exhaustification is highly restricted in downward-entailing environments). We could analyze *I'll invite Ann or Bill* in one of two ways: as being systematically associated with the exhaustivity operator O in the syntax, as in (2); or as never being associated with O, but with a revised semantics that interprets *A or B* in essence as *only [A or B]*, as in (3). The semantic result is the same, although the claim about the syntax isn't. The point is straightforward: if O is systematically present, there is no need for a symbol to 'tell' the semantics about its presence.

Notation: We write the semantic value of an expression *F* as **F**. *S* is any expression of propositional type.

(1) I'll invite Ann or Bill, or schematically: A or B

(2) **Analysis with O**

Logical Form: O [A or B]

Meaning:

$[\![\text{A or B}]\!]^w = \mathbf{A}(w) \text{ or } \mathbf{B}(w)$

$[\![\text{O S}]\!]^w = [\mathbf{O}(\lambda w'\, [\![S]\!]^{w'})](w)$

$[\![\text{O [A or B] }]\!]^w = [\mathbf{O}\, (\lambda w'\, [\![\text{A or B }]\!]^{w'})](w)$

$= [\mathbf{O}\, (\lambda w'\, [\mathbf{A}(w') \text{ or } \mathbf{B}(w')]](w)$

(3) **Analysis without O**

Logical Form: A or B

Meaning:

$[\![S]\!]^w = [\mathbf{O}(\lambda w'\, [\![S]\!]^{w'})](w)$

$[\![\text{O [A or B] }]\!]^w = [\mathbf{O}\, (\lambda w'\, [\![\text{A or B }]\!]^{w'})](w)$

$= [\mathbf{O}\, (\lambda w'\, [\mathbf{A}(w') \text{ or } \mathbf{B}(w')]](w)$

In actual fact, exhaustification is notoriously constrained: it doesn't normally weaken the meaning of the target expression and is thus dispreferred in downward-entailing environments, contrary to our fanciful analysis of O. But if the semantics of O were so environment-sensitive as to be able to be 'turned off' in negative environments, as argued for instance by Magri REF, O could be inserted everywhere—and thus it could be integrated to the semantic procedure. Precisely this conclusion has been reached about A, and also about O itself, in recent research.

1.2 The story of A

When a presupposition fails to be satisfied, as in (4), two repair strategies can be used (e.g. Heim 1983).

(4) The king of Syldavia didn't come.

Using global accommodation, the addressee computes the presupposition as required by presupposition projection rules, sees that in view of her beliefs the sentence should give rise to a failure, and adapts her beliefs to avoid this outcome—thus adopting the assumption that Syldavia has a king. But there is also another option, which is made necessary in a discourse such as: *The king of Syldavia didn't come because there is no king of Syldavia*. For the addressee to adopt the assumption that Syldavia has a king wouldn't help because it would lead to a contradiction. Heim 1983 proposes that, at some cost, a presupposition can be locally accommodated in the scope of operators. Without going into details of implementation, this operation accommodates a presupposition in the scope of operators, and has the effect of turning it into part of the assertive component at the level of a constituent. It wasn't presented by Heim 1983 as being syntactically represented, but rather as a pragmatic repair strategy.

It was later proposed that local accommodation can be analyzed by way of the insertion of an optional operator that turns undefinedness, notated as #, into falsity, notated as 0, as in (5) (Beaver 2001, Beaver and Krahmer 2001, Fox 2013). This operator, originally due to Bochvar 1939, can be applied to a variety of trivalent logics, including to the trivalent core of Heim's dynamic semantics.

(5) *AF* has the value 1 iff *F* has the value 1; otherwise, *AF* has the value 0.

Chatain and Schlenker 2023a object that when ellipsis tests are applied, as in (6), it becomes very dubious that A is syntactically real.

(6) *Context:* We're supposed to take the lab rat out of its cage, once a day. Otherwise, it feels stressed. Bill has been unreliable in performing this task.

Ann: Last Monday, Bill didn't take the lab rat out of its cage.
(⇝ last Monday, the rat was initially in its cage)
Sue: On Wednesday as well, but that's just because I took it home on Tuesday and forgot to bring it back, so it wasn't in the cage at all that day.

Ann's statement gives rise, despite the negation, to an inference that last Monday the lab rat was in the cage. This suggests that *take out of its cage* triggers the presupposition that the rat was in the cage, and crucially that no occurrence of A appears below negation, hence a Logical Form such as (7)a below. But under standard assumptions about ellipsis, the elided clause (uttered by Sue) is recovered by copying the boxed antecedent. Since this boxed constituent doesn't contain A, neither does the elided clause. This predicts that the presupposition of the elided clause cannot be locally accommodated, which should give rise to a contradiction in view of the *because*-clause. Since no such contradiction is obtained, local accommodation must be applied in the elided clause—but this seems to argue against the operator-based account.

(7) a. X not PP'. Y too <not PP'> (because not Y P).
b. Impossible LF:
X not PP'. Y too <not **A** PP'>.

Chatain and Schlenker 2023a develop an operator-free analysis modeled after domain restriction. In the first sentence of (8)a, *every professor* means *every MIT professor*, so that the semantic value λx x is a professor is in essence strengthened by a conjunct into: λx x is at MIT and is a professor. Importantly, this strengthening operation is environment-dependent. This is shown by the fact that the elided VP *meet every professor* in the second sentence of (8)b means *every Harvard professor*. This dependency can be analyzed by positing the functional domain restriction *C(x)* in (8)c, where x is a bound variable.

(8) a. MIT's dean met with every professor. Harvard's dean didn't.
b. **Simple parse:**
MIT's dean met with every$_C$ professor. not Harvard's dean <~~met with every$_C$ professor~~>.
c. **Functional parse:**
MIT's dean λx x met with every$_{C(x)}$ professor. not Harvard's dean will <λx x ~~met with every$_{C(x)}$ professor~~> too.

As Chatain and Schlenker 2023a argue (following earlier literature), in the general case such domain restrictions can depend on an arbitrary number of variables. Generalizing 'to the worst case', they make them dependent on an entire assignment function, and do so without positing any additional symbols in Logical Forms. They then propose to extend this mechanism from the nominal to the verbal case, calling it 'Generalized Domain Restriction'. The key idea is that, at some cost, the predicative meaning *λx x takes the rat out its cage* can be interpreted as *λx <u>x has exactly one salient dog in cage</u> and x takes the rat out its cage*. The first conjunct justifies the presuppositions of the second, which yields the same effects as local accommodation. Importantly, the operation isn't encoded in the syntax (these particular λ-expressions are meanings, not forms), but is so environment-dependent (technically: dependent on parameters of the interpretation function) that one and the same VP can obtain different domain restrictions in different environments. This, in turn, solves the problem of apparent non-parallelism under ellipsis. The key is that the two VP meanings are in fact parallel, but they are functional on different parameters and thus can get different domain restrictions.

Chatain and Schlenker 2023a opt for an operator-free analysis for reasons of simplicity and elegance. But they could just as well have posited an operator, say A*, that is present everywhere but adds domain restrictions in an environment-sensitive fashion (depending on parameters of the interpretation function).

1.3 The story of O

Chatain and Schlenker 2023b reach a related conclusion in a somewhat different way in an analysis of O, the exhaustivity operator. While they consider similar ellipsis-based tests as those discussed for A, they deem them inconclusive because the sheer complexity of the crucial examples makes them very difficult to assess. On the other hand, they argue that tests based on propositional and predicative anaphora suggest that the semantics of O is extremely powerful, and can be 'turned off' in an environment-sensitive fashion.

As is well-known, Chierchia 2004 argued that some scalar implicatures are computed locally, in the scope of various logical operators.[1] Chierchia originally revised the interpretive procedure so as to allow scalar implicatures to be computed, operator-free, in tandem with compositional interpretation. Soon after, however, the same facts were handled by keeping the semantic procedure constant while postulating a covert operator O, which as a first approximation is a presupposition-less version of *only* (e.g. Chierchia et al. 2012).

While there are multiple arguments for O, one of the most convincing pertains to Hurford's constraint, the observation that in configurations such as (9)a, deviance is obtained if the second disjunct entails the first. But when a scalar term appears in the first disjunct, as in (9)b, the constraint is apparently obviated. This can be explained if the constraint is still in force, but the meaning of the first disjunct has been strengthened by a local implicature implemented through O, as in (9)c; in its presence, the first disjunct means in essence *Ann only read some of the books*, and it is not entailed by the second disjunct any longer.

[1] Landman 2000 and Schwarz 2001 independently explored related ideas.

(9) a. #Ann lives in France or in Paris.
b. Ann read some of the books or she read all the books.
c. O [Ann read some of the books] or she read all of the books.

Chatain and Schlenker 2023b consider the interaction between anaphora and exhaustification, and find that readings that are predicted to arise are missing. The simplest case appears in (10)a: the first sentence can naturally be read with exhaustification of the disjunction, but even when this is the case the second sentence can only access its unstrengthened meaning; using the schematic representation in (10)b, *it* can refer back to *[S or P]*, not to *O [S or P]*.

(10) a. Syldavia will invade Spain or Portugal. However Biden doesn't believe it.
b. O [S or P]. not B believe it.

A related problem arises in cases of Hurford disjunction, as in (11). In (11)a, an exhaustivity operator must be present in the first disjunct, as represented in (11)b, because otherwise the second disjunct would entail the first, in violation of Hurford's constraint. TO BE ADAPTED But in the continuation in (11)c, 'the former' cannot access the exhaustified meaning of the first disjunct: (11)c entails that the director won't invite the first-year students, whether alone or with others (the same facts hold in (11)d, whose anaphoric structure is a bit less transparent). By contrast, when an overt *only* is used in the first disjunct, as in (12), the facts change, and *the former* preferentially targets the exhaustified version of the first disjunct.

(11) a. The assistant director will invite the first-year students, or all of the students.
b. Assistant-Director λx. [O Few x] or [All x]
c. The director will do neither the former nor the latter.
d. The director will do neither.

(12) The assistant director will invite only the first-year students, or all of the students. The director will do neither the former nor the latter / neither.

Chatain and Schlenker 2023b conclude that in their examples, scalar terms display a Janus-faced behavior: their exhaustified value contributes to the truth conditions; but only their unstrengthened value is accessed by anaphora. They conclude that there must be some way to retrieve the unstrengthened meaning from the enriched meaning of a constituent F enriched by O: even if *it* in (10)a or *the former* in (11)c is coindexed with the strengthened constituent *[O F],* the effect of O can be turned off in some environments, notably negative ones. After discussing various technical ways to achieve this, they draw a more general conclusion about the syntax/semantics interface: "since the semantics of the exhaustivity operator must be flexible enough that it can in principle be inserted everywhere (without necessarily having semantic effects), this opens the possibility that whatever semantic effect O has can be directly built into the semantic procedure. This would yield an operator-free approach to exhaustification".[2]

1.4 Outlook and structure

In sum, recent research has argued that the accommodation operator A is not syntactically real, and that the exhaustivity operator O has such a context-sensitive semantics that it can be inserted at all relevant sites in the syntax with effects that can be turned off—with the result that it can just as well be incorporated, operator-free, to the semantic procedure itself.

But there is a new kid on the block: in a highly thought-provoking proposal, Blumberg and Goldstein (2021a,b) have argued that apparently pragmatic conditions of non-triviality should be captured by a non-redundancy operator R, loosely modelled after O.[3] We will outline the case for R,

[2] We have replaced *Exh* with *O* in this quote; nothing hinges on this notational difference.
[3] As we understand it, Blumberg and Goldstein are working on more sophisticated versions of the R-based theory. We only discuss the version they presented in talks and manuscripts cited in the references.

argue that ellipsis-based tests suggest that the operator is not syntactically real, and derive Blumberg and Goldstein's main data through operator-free means.

The rest of this piece is organized as follows. We start by summarizing the case for R (Section 2). We then discuss and criticize its application to diversity inferences, and present an alternative in terms of lexical presuppositions (Section 3). We then turn to the application of R to ignorance inferences and raise problems for it (Section 4), before developing an operator-free pragmatic alternative in terms of Stalnakerian anti-triviality conditions, first applied to embedding under *believe* (Section 5), and then to embeddin g under *hope* (Section 6). An appendix discusses the application of R to free choice inferences, and argues that here too R can be dispensed with owing to the existence of independently motivated accounts of free choice.

2 The case for R

2.1 General picture

To introduce Blumberg and Goldstein's initial motivation, we note that a disjunction *A or B* is typically deviant unless the context leaves open that *A* might be true or false, and similarly for B. Blumberg and Goldstein 2021a,b argue that such non-triviality conditions sometimes permeate at-issue truth conditions upon embedding, as in (13) (see also also Cremers et al. 2019).

(13) *Context:* There are three detectives: one has already ruled out Ann and is certain that Bill is the culprit, but the others don't know anything yet.

Exactly two detectives believe/hope/fear that Ann or Bill committed the crime.
(Blumberg and Goldstein 2021a)

An accurate paraphrase of the truth conditions seems to be: exactly two detectives (i) believe that at least one of Ann and Bill committed the crime, and (ii) leave open that Ann might or might not be the culprit, and similarly for Bill. Without (ii), it wouldn't be true that exactly two detectives satisfy the condition, since all three satisfy (i). In other words, the non-triviality conditions that apply to an utterance of *A or B* seem to affect the truth conditions of the belief report.

In this and a series of rather different cases, Blumberg and Goldstein argue that a non-redundancy operator R can appear in Logical Forms. For reasons we will come to later, they posit that in the positive case, the operator applies to the first disjunct of the embedded clause, yielding the simplified representation in (14)a. In the negative case, displayed in (14)b, the operator is not inserted at all. Finally, the desired reading of (13) is derived with the representation in (14)c, which again contains the non-triviality operator R.

(14) a. The detective believes that R[Ann <committed the crime>] or Bill committed the crime.
Paraphrase: The detective believes that at least one of Ann and Bill committed the crime, and leaves open whether Ann might or might not have committed the crime.
b. The detective doesn't believe that [Ann <committed the crime>] or Bill committed the crime.
Paraphrase: The detective doesn't believes that at least one of Ann and Bill committed the crime,
c. Exactly two detectives believe that R[Ann <committed the crime>] or Bill committed the crime.
Paraphrase: Exactly two detectives believe that at least one of Ann and Bill committed the crime, and leave it open whether Ann might or might not have committed the crime.

The presence of R can yield readings that do not entail the corresponding R-free sentence. Such is the case in (14)c, in the scenario in (13): the sentence with R is true but the sentence without R is false because all three detectives believe (the literal meaning of) the disjunction.

Our main argument will be that when ellipsis tests are applied, the syntactic reality of R becomes dubious because of examples such as (15).

(15) *Context:* There are three detectives: one has already ruled out Ann and is certain that Bill is the culprit, but the others don't know anything yet.
a. Exactly two detectives believe that Ann or Bill committed the crime, but no journalist believes that

Ann or Bill committed the crime.
b. Exactly two detectives believe that Ann or Bill committed the crime, but no journalist does.

Due to ellipsis, the second conjunct of (15)b must be identical to the first with respect to the presence or absence of A. Not having A at all would make the first conjunct false in view of the context. Including A, as in (16)b, predicts that the sentence with ellipsis just couldn't get the reading that the sentence without ellipsis does. This seems to us to be an incorrect prediction.

(16) a. LF of (15)a
Exactly two detectives believe that R [Ann <committed the crime>] or Bill committed the crime, but no journalist believes that Ann or Bill committed the crime.
b. LF of (15)b
Exactly two detectives believe that R [Ann <committed the crime>] or Bill committed the crime, but no journalist does ~~believe that R [Ann <committed the crime>] or Bill committed the crime~~.

We will run the same tests on other purported uses of R, and reach in several cases the same conclusion: this operator is not syntactically real.

2.2 Data types

Blumberg and Goldstein's 2021a,b use R to derive three kinds of enrichments that are computed in the scope of operators: diversity inferences, ignorance inferences and free choice inferences, which are exemplified in (17).

(17) a. Diversity inferences
The detective hopes/fears that the surveillance cameras are functional.
⇒ the detective isn't certain whether the surveillance cameras are functional.
b. Ignorance inferences
The detective believes Mary or Sue committed the crime.
⇒ the detective doesn't know whether Mary or Sue committed the crime.
c. Free choice inferences
Jim can eat ice-cream or cake.
⇒ Jim can eat ice-cream and Jim can eat cake.

Diversity inferences refer to cases in which embedding of a clause *F* under an attitude verb such as *hope* or *fear* triggers an inference that the attitude holder believes *F* to be possibly true and possibly false, as illustrated in (17)a. Embedded ignorance inferences refer to cases in which a disjunction *A or B* (or sometimes conjunction *A and B*) embedded under an attitude verb triggers the inference that the attitude holder takes the status of *A* and of *B* to be open, as illustrated in (17)b. Free choice inferences refer to cases in which a disjunction embedded under a possibility modal, as in (17)c, triggers the inference that each disjunct is possible.

As already illustrated in (16), Blumberg and Goldstein observe that these meaning components can sometimes make at-issue contributions in the scope of various quantifiers, just as has been argued for local exhaustification. And just as exhaustification in the scope of operators provides a key argument for O, non-redundancy computed in the scope of operators provide a key argument for the non-redundancy operator R.

We will now cast doubt on the syntactic reality of R. We start by showing that the inferences Blumberg and Goldstein consider are not all of the same nature. We propose that diversity conditions are lexically triggered presuppositions and not the by-product of a general non-redundancy mechanism. After excluding diversity inferences from consideration, we will apply the ellipsis test to show that ignorance inferences cannot be syntactic enrichments, as proposed by Blumberg and Goldstein, and we will propose an alternative analysis in terms of *pragmatic* non-triviality conditions. We will not discuss Free Choice inferences in any detail because they have been the object of numerous accounts in the literature; but in the Appendix, we suggest that whatever positive features there are in Blumberg and Goldstein's R-based account of Free Choice can be retained in an operator-free analysis.

3 Diversity Inferences with and without R

We start by summarizing the R-based account of diversity inferences, and then argue that these are better analyzed as simple lexically encoded presuppositions.

3.1 Diversity inferences with R

Blumberg and Goldstein observe that a number of attitude verbs, such as *hope*, *fear* and *wonder*, come with diversity conditions: they can only be used if the attitude holder believes the embedded proposition to be possibly true and possibly false, as illustrated in (18).

(18) a. *Context:* The detective believes that Ann didn't commit the crime.
#He hopes that/fears that/wonders whether she did.
b. *Context:* The detective believes that Ann committed the crime.
#He hopes that/fears that/wonders whether she did

In Blumberg and Goldstein's proposal, this diversity requirement arises from a condition against redundant material, reified by way of a syntactic operator R. Simply put, R yields undefinedness if its propositional argument is uniformly true or uniformly false throughout the local context. A formal definition is given in (19)a, where lc is a local context parameter (something we do not make use of in our own treatment). The proposal also relies on the two auxiliary assumptions listed in (19)b. Assumption (i) specifies how the local context of the embedded clause is computed. Assumption (ii) states that semantic failure in the embedded clause does not 'project' to the embedding clause, but rather yields falsity.

(19) a. **Semantics of R**
$[\![RA]\!]^{c, s, lc, w} = \#$ unless for some w', w" in lc, $[\![A]\!]^{c, s, lc, w'} \neq [\![A]\!]^{c, s, lc, w''}$; if $\neq \#$, $= 1$ iff $[\![A]\!]^{c, s, lc, w} = 1$.

b. **Auxiliary assumptions**
(i) The attitude verbs in (17)-(18) set the local context of the embedded clause to the doxastic alternatives of the matrix agent.
(ii) The semantics of these attitude verbs is bivalent: failure of the embedded clause yields falsity of the embedding clause.

With the LF in (20)a below, and the auxiliary assumptions, this proposal correctly predicts the diversity inferences for *hope* seen in (18).

(20) a. x hopes Ra
a'. **Predicted truth conditions**
In the best of x's doxastic alternatives, *A* is true, in some of x's doxastic alternatives, *A* is true, while in others, *A* is false

An R-based account also predicts that diversity inferences can be generated in the scope of various quantifiers. Blumberg and Goldstein give the following example to illustrate:

(21) *Context:* There are three detectives and several suspects. All three detectives most desire that Ann committed the crime, since they already have her in custody. One detective is sure that Ann did it, but the others don't know anything yet.
Exactly two detectives hope that Ann committed the crime

As stated, however, the account of diversity inferences suffers from one empirical and one conceptual problem, requiring some refinements. Empirically, the case of *believe* is problematic. *Believe* does not trigger a diversity inference, as such an inference would contradict what is asserted by the sentence: in (22)a, the assertion rules out that the detective leaves open whether Mary is the culprit.

(22) a. The detective believes that Mary is the culprit.
⇏ The detective does not believe that Mary is the culprit.
b. The detective believes R [Mary is the culprit].

This shows the need to regulate the distribution of R. To rule out (22)b and other distributional requirements, Blumberg and Goldstein 2021a invokes the principles of Contradiction Avoidance and the Strongest Meaning Hypothesis, described below. To avoid generating contradictions, they also assume that Contradiction Avoidance overrides the Strongest Meaning Hypothesis.

(23) **Contradiction Avoidance:** Do not insert R if inserting it makes the sentence contradictory.
Strongest Meaning Hypothesis: When choosing between parses of a sentence that only differ in the distribution of the R operator, pick that parse which yields the strongest meaning.

In the later piece Blumberg and Goldstein 2021b, the authors issues with the principles in (23) and leave for future research a more detailed investigation of the distribution of R. (Our arguments in the sequel will not rely on the specifics of the distribution of R, and we may keep the principles in (23) as a useful guide in distinguishing parses.)

A further problem is that, conceptually, Blumberg and Goldstein's assumption (in (19)b(ii)) that attitude verbs converts failure of the embedded clause to falsity (of the embedding clause) seems at odds with the fact that attitude verbs project presuppositions. In fact, they assume that the failure triggered by R, notated as #, is not presupposition failure, but another kind of failure living alongside presupposition failure, hence a somewhat complicated account.

3.2 Diversity conditions as simple presuppositions

Having spelled out Blumberg and Goldstein's basic account, we turn to some challenges. In their proposal, diversity inferences are derived through a uniform mechanism, namely the insertion of the operator R. It derives diversity inferences as part of the assertive content: while the semantics of R is trivalent, the attitude verb makes its contribution at-issue (per the assumption in (19)b(ii)).

However, diversity inferences triggered by different attitude verbs display different pragmatic statuses. To start with a simple case, the diversity inferences of *wonder* don't project over negation or in questions, as illustrated in (24), and in this respect they behave like ordinary at-issue content. This is as expected on the R-based account.

(24) a. Does Jane wonder whether it's raining?
⇏ it's not the case that Jane believes it's raining
b. Jane isn't wondering whether it's raining.
⇏ it's not the case that Jane believes it's raining

Turning to *hope*, Blumberg and Goldstein claim that its diversity inferences likewise do not project from these environments ; in particular, they claim that neither (25)b nor (25)c give rise to an inference about Jane's belief about the weather.

(25) a. Jane hopes that it's raining.
⇒ it's not the case that Jane believes it's raining
b. Jane doesn't hope that it's raining
⇒ it's not the case that Jane believes it's raining(?)
c. Does Jane hope that it's raining?
⇒ it's not the case that Jane believes it's raining (?)

But here we think these sentences do in fact give rise to such an inference. It's possible to make the presence of this inference felt by considering cases of unknown identity. Suppose we turn on the TV to a certain channel. There are two contestants, whose names we don't know. The contestants have a card in front of them ; if the card is red, they win a generous voucher for a bookstore; if the card is black, they win a free massage in the spa. One contestant has already turned their card face up and discovered that it is black; the other contestant's card is still face down. A commenter makes the following statement:

(26) A. Does Jane hope that her card is black?
B. If Jane hopes that her card is black, she will likely be very happy.

It seems to us that Jane is likely the contestant who hasn't turned over a card yet. That can only follow if these sentences yield the inference that Jane does not believe her card is black, something that is only true of the contestant with the face-down card. In short, it does seem that contra Blumberg and Goldstein, the diversity inference does project.

Blumberg and Goldstein propose another argument. They note that it is possible for a speaker to assert their ignorance about Jane's belief state and then proceed with one of the two clauses in (25)b and (25)c, as in the examples in (27). This should not be possible if these clauses projected the inference that Jane does not believe that it is raining.

(27) a. I have no idea what Jane thinks of the weather, but if she hopes it's raining, she's going to be disappointed.
b. I have no idea what Jane thinks of the weather but does she hope it's raining?

This, they claim, stands in contrast with what is found with soft presupposition triggers, which cannot be preceded by a clause that expresses ignorance about the relevant presupposition, as illustrated in **Error! Reference source not found.**:

(28) a. #I have no idea if Jane has ever smoked but did she stop smoking?
b. #I have no idea if Jane has ever smoked but if she stopped smoking, then Bill has too

However, we find that the contrast is not as strong as is reported. With proper narration, the sentences with soft triggers become quite acceptable, as illustrated in

(29) *Context:* Jane has been quite nervous and angry for the past couple of weeks. I barely know her but I speculate.
a. I have no idea if Jane smoked, but did she stop smoking recently?
b. I have no idea if Jane smoked but if she stopped smoking recently, then she's probably in withdrawal now.

In the case of soft presupposition triggers, these sentences are acceptable because the projection of the inference may be prevented via the process of local accommodation (see Section 5.5). This process does require some pragmatic set-up and this may be the reason why the initial sentences provided by Blumberg and Goldstein did not sound felicitous. We speculate that local accommodation of the diversity inference is the mechanism by which the sentence with *hope* come out acceptable too.

In conclusion, we take it that the TV show scenario above shows that *hope* does trigger diversity inferences that project in these environments, as seen in (26).[4] This is unexpected for Blumberg and Goldstein's account: in their account, all diversity inferences are at-issue hence should fail to project.

The observed heterogeneity of diversity inferences is unexplained if one and the same mechanism, namely the R-operator, underlies them all. We find the following alternative explanation

[4] We don't include the other part of the diversity inference: *it's not the case that Jane believes it's not raining*. We believe that this inference may be at-issue for *hope*. For instance, a sentence such as *Jane no longer hopes to win the election* seems (to our ear) acceptable in a context where Jane has just learned that she has lost. Here, no inference is triggered of the form: *not [Jane believes she hasn't won]*. For simplicity, we disregard this point in our discussion of Blumberg and Goldstein's account, but we do take it into account in our lexical entry for *hope* in (30) (where the condition that the attitude holder takes the embedded proposition to be possible is at-issue). In the end, *wonder*, *hope* and *want* all display slightly different behaviors with respect to these diversity inferences: for *wonder*, they are at-issue; for *want*, they are presuppositional; for *hope*, one is presuppositional and one is at-issue. This typology only reinforces our point that lexical idiosyncracies are at work here.

more natural: diversity inferences are part of the lexical semantics of the attitude verb.[5] Specifically, we propose a lexical entry for *hope*, in (30), which has a diversity inference as a lexical presupposition, while the lexical entry for *wonder whether*[6], in (31), encodes it as an assertion. Both entries draw from Heim (1992), who gave a related analysis for *want*, reproduced in modified form in (32).[7] (From here on, we don't explicitly consider in our lexical entries cases in which arguments of an item may evaluate to #; we assume that an explanatory theory of presupposition projection explains how undefinedness in the arguments translate to undefinedness in the output.)

(30) A Heim-inspired semantics for *hope*
$[\![\text{hope}]\!]^{c,s,t,w}(p)(x) = \#$ iff $\mathbf{Dox_w(x) \cap \{w': p(w') = 0\} = \emptyset}$;
$[\![\text{hope}]\!]^{c,s,t,w}(p)(x) = 1$ iff $\mathbf{Dox_w(x) \cap \{w': p(w') = 1\} \neq \emptyset}$ and $\text{Best}(\text{Dox}_w(x)) \subseteq \{w': p(w') = 1\}$

(31) A Heim-inspired semantics for *wonder whether*
$[\![\text{wonder whether}]\!]^{c,s,t,w}(p)(x) = 1$ iff $\mathbf{Dox_w(x) \cap \{w': p(w') = 1\} \neq \emptyset}$ and $\mathbf{Dox_w(x) \cap \{w': p(w') = 0\} \neq \emptyset}$
$[\![\text{wonder whether}]\!]^{c,s,t,w}(p)(x) = 0$ otherwise

(32) A Heim-inspired semantics for *want* (modified from Heim 1992 to state the at-issue conditions in terms of Best, as in Blumberg and Goldstein 2021a,b)
$[\![\text{want}]\!]^{c,s,t,w}(p)(x) = \#$ iff $\mathbf{Dox_w(x) \cap \{w': p(w') = 1\} = \emptyset}$, **or** $\mathbf{Dox_w(x) \cap \{w': p(w') = 0\} = \emptyset}$;
$[\![\text{want}]\!]^{c,s,t,w}(p)(x) = 1$ iff $\text{Best}(\text{Dox}_w(x)) \subseteq \{w': p(w') = 1\}$

A harder challenge for this lexicalist account is to explain Blumberg and Goldstein's observation that diversity inferences (be they presuppositional as with *hope* or assertive as with *wonder whether*) may sometimes intrude into at-issue truth conditions under quantifiers, as in (33), repeated from (21).

(33) *Context:* There are three detectives and several suspects. All three detectives most desire that Ann committed the crime, since they already have her in custody. One detective is sure that Ann did it, but the others don't know anything yet.
Exactly two detectives hope that Ann committed the crime

Fortunately, there are two reasonable solutions under our lexical account. First, the intrusive reading could be handled as a case of local accommodation of the diversity presupposition of *hope*. Second, local accommodation might not even be necessary if presuppositions are projected in existential form by numerals, as is suggested by the experimental results of Chemla 2009. Under the 'existential projection' view, the lexical entry for *exactly two* might be that in (34). It has a very weak presuppositional requirement, to the effect that at least one of the three detectives takes it to be possible that Ann didn't commit the crime. This existential presupposition is satisfied by the scenario of (33). The at-issue truth conditions end up counting the number of detectives that satisfy the ('possibly false') presupposition combined with the at-issue contribution of the verbal predicate, which yields the desired result.

(34) **Existential projection rule for *exactly two***
If **NP** and **VP** are of type <e, t>,

[5] For theories that posit a general algorithm to trigger apparently lexical presuppositions in other cases, these cases would need to be revisited as well.

[6] This is a shortcut: any appropriate semantics for *wonder whether* should derive its meaning compositionally, from a denotation for *wonder* and the meaning of an indirect yes-no question. We won't pursue this point here.

[7] As a reviewer notes, the diversity inference of *want* is not so robust. The sentence *I want this weekend to last forever* seems acceptable, whereas the sentence "*#I hope this week-end lasts forever*" isn't, as first noted by Heim (1989). The reviewer notes that the variability is in line with an operator-based approach to non-redundancy, if the operator is optional. But an optional operator would make diversity inference optional in all cases, even with *hope*, thus failing to explain any contrast. We leave an account of this variability in robustness to future research.

⟦Exactly two⟧ (**NP**)(**VP**) ≠ # only if for at least one object d, **NP**(d) = 1 and **VP**(d) ≠ #;
if ≠ #, ⟦Exactly two⟧ (**NP**)(**VP**) = **1** iff |{d: **NP**(d) = **VP**(d)}| = 2

Owing to the heterogeneous behavior of diversity inferences, we conclude that a lexical account of diversity inferences is empirically more adequate than the R-based analysis. It is also more parsimonious, as it does not rely on the postulation of a new covert operator.

This conclusion leaves open the possibility that Blumberg and Goldstein's account might be correct for the other inferences they discuss, in particular ignorance inferences. But in the next sections, we show that the ellipsis test casts doubt on the syntactic reality of R, a result that dovetails with Chatain and Schlenker's (2023b) discussion of the accommodation operator A. We propose instead an entirely pragmatic account of Blumberg and Goldstein's ignorance inferences, based on older Stalnakerian ideas.

4 Ignorance Inferences with R

Blumberg and Goldstein's embedded ignorance inferences involve cases in which *A or B* as well as *A and B* are embedded under attitude verbs, and yield the inference that the attitude holder is ignorant about the status of *A*, *B*. Blumberg and Goldstein propose to treat such inferences as instances of non-redundancy implemented in terms of the R operator discussed in the previous section. The same ellipsis-based tests as in our discussion of the operator A will argue against this syntactic treatment, and in favor of an operator-free alternative.

4.1 The case for R

Blumberg and Goldstein 2021a,b argue that (35) yields the inference that the agent is ignorant about the individual disjuncts.

(35) The detective believes that/hopes that/fears that/wonders whether Ann or Bill committed the crime.
=> the detective thinks it's possible that Ann committed the crime
=> the detective thinks it's possible that Bill committed the crime

Blumberg and Goldstein also show that with all attitudes considered but *believe*, similar ignorance inferences are triggered by conjunctions, as illustrated in (36).

(36) Mary hopes that/fears that/wonders whether Ann brought apple pie and Bill brought blueberry pie.
=> Mary thinks it's possible that Ann didn't bring apple pie
=> Mary thinks it's possible that Bill didn't bring blueberry pie

Blumberg and Goldstein's analysis relies on the insertion of R in appropriate positions. They posit representations such as those in (37)-(38). A key auxiliary hypothesis, which is standard in local context theory, is that in *[A and B]* the local context of *B* is the local context of the entire embedded clause updated with *A*, whereas in *[A or B]* the local context of *B* is the local context of the embedded clause updated with *not A*; this is stated in (38).

(37) a. The detective hopes that Ann or Bill committed the crime.
b. x hopes RA ∨ RB

(38) a. Mary hopes Ann brought apple pie and Bill brought blueberry pie.
b. x hopes RA ∧ RB

(39) Auxiliary assumptions
a. In *[A and B]* the local context of *B* is the local context of the entire constituent updated with *A* (i.e. intersected with the value of *A*).
b. In *[A or B]* the local context of *B* is the local context of the entire constituent updated with *not A* (i.e. intersected with the value of *not A*).

The structures in (37)b and (38)b deliver the correct ignorance inferences. As mentioned in the previous section, Blumberg and Goldstein take the local context of the embedded clause to be the set of doxastic alternatives of the agent. With the assumptions in (39), these principles and the semantics of R entail that *[R A]* is defined only if *A* is undecided over the set of doxastic alternatives, i.e. the detective deems *A* and its negation to be possible. In (38), *[R B]* will be defined only if *B* is undecided over the set of A-worlds among the agent's doxastic alternatives. Similarly, in (37), *[R B]* will be defined only if *B* is undecided over the set of not-A-worlds among the agent's doxastic alternatives. In particular, we have derived the desired ignorance inferences: disjunction embedded under attitudes will only be felicitous if each disjunct is true in some doxastic alternative, and conjunction embedded under attitudes will only be defined if each conjunct is false in some doxastic alternative.

As Blumberg and Goldstein point out, the structures in (37)b-(38)b in fact yield stronger inferences. In the case of disjunction, it is also required that some doxastic alternatives satisfy neither *A* nor *B*. Otherwise, *A or B* would hold across these alternatives, and thus *B* would be implied by *[not A]*, with the result that *B* would be redundant in its local context. This is simply the diversity inference derived in a different manner. As seen in the previous section, such an inference will be contradictory in the case of *believe*. In other words, the structure in (40)c is not admissible:

(40) a. The detective believes that Ann or Bill committed the crime.
b. x believes RA ∨ B
c. #x believes RA ∨ RB

Blumberg and Goldstein rule out (40)c using Contradiction Avoidance as stated in (23). Fortunately, the structure in (40)b is sufficient to generate the ignorance inference: it implies that the agent does not know whether *A*; and combined with the fact that she believes *A or B*, this implies that she believes *B* is possible. [8]

As in the previous section, the syntactic nature of Goldstein and Blumberg's account predicts that ignorance inferences could make a contribution in the scope of quantifiers, a case illustrated in (41) and predicted by the structures in (42).

(41) a. *Context:* There are three detectives, and two possible suspects: Ann and Bill. One detective has already ruled out Ann, but the others haven't ruled out either Ann or Bill.
Exactly two detectives believe that Ann or Bill committed the crime.

b. *Context:* There are three detectives and three possible suspects: Ann, Bill, and Carol. The detectives have Ann and Bill in custody, but can't find Carol. One detective is sure that Ann didn't do it, but the others don't know anything yet.
Exactly two detectives hope that Ann or Bill committed the crime.

c. *Context:* Three of Mary's friends are at a potluck dinner. All three most prefer apple pie and blueberry pie to any other type of pie. One already knows that Ann brought apple pie, but the others don't know anything about who brought what.
Exactly two of Mary's friends hope that Ann brought apple pie and Bill brought blueberry pie.

(42) a. [Exactly two detectives] $\lambda x\ t_x$ believe [RA or B]
b. [Exactly two detectives] $\lambda x\ t_x$ hope [RA or RB]
c. [Exactly two detectives] $\lambda x\ t_x$ hope [RA and RB]

4.2 Problems

Despite these positive results, we will now show that the R-based account of these ignorance inferences makes incorrect predictions about ellipsis, and we will offer an operator-free alternative analysis. In Section 3, we argued that the lexical semantics of attitudes verbs is responsible for their diversity inferences. But this account cannot extend to ignorance inferences triggered by embedded conjunctions

[8] Still, this does not imply that *[not B]* is possible. This account thus falls short of deriving true ignorance inferences. This problem also befalls the account we will later present.

and disjunctions, as the latter are not directly visible to the lexical semantics of the attitude verbs. We will thus develop a different account based on local pragmatic principles.

4.2.1 Problems with ellipsis: believe

Blumberg and Goldstein's syntactic analysis makes a prediction: in ellipsis, if the elided material is large enough, R should be present in the elided expression as well as in its antecedent, or it should be absent from both, as schematically illustrated in (43), where we once again enclose the elided material within angle brackets.

(43) a. x $V_{attitude}$ [RA or (R)B]. Not y <$V_{attitude}$ RA or (R)B>
b. x $V_{attitude}$ [A or B]. Not y <$V_{attitude}$ A or B>

These predictions appear to be in error. Consider the dialogue in (44). If A's utterance has the structure in (43)a, with R, then B's utterance should contain R as well. The presence of R in B's utterance generates an ignorance inference in the scope of the attitude verb ; because of the semantics of attitude verb (as in (19)b(ii)), this inference becomes at issue can thus be targeted by negation. In consequence, either of the continuations listed should be felicitous, contrary to fact. If A's utterance has the structure in (43)b, this problem does not arise, but another one does: A's assertion is incorrectly predicted not to trigger ignorance inferences.

(44) Dialogue:
A: The detective believes that Ann or Bill committed the crime.
B: I don't!
(#In fact, I know that Bill did it.)
(#In fact, I know that Ann did it.)

Related problems arise with conjunction. To account for the ignorance inferences triggered by each conjunct, Blumberg and Goldstein must analyze (45)a as (45)a' and (45)b as (45)b': in each case, *R* appears twice in the first embedded clause, which is then copied under negation in the second clause. This predicts readings on which the negation could be true simply because the speaker is certain that Ann will or did bring apple pie. While this might be a possible reading, this does not seem to us to be the only reading. Specifically, the salient reading of (45)a implies that I don't want Ann to bring apple pie and I don't want Bill to bring blueberry pie, but the inference doesn't follow from the R-full representation in (45)a'.

(45) a. Mary wants Ann to bring apple pie and Bill to bring blueberry pie, but I don't.
a'. x wants [RA and RB]. I don't <want [RA and RB]>.
b. Mary hopes that Ann brought apple pie and Bill brought blueberry pie, but I don't.
b'. x hopes [RA and RB]. I don't <hope [RA and RB]>.

4.2.2 Problems with projection: want *and* hope

Under *want*, the R-based account fails to predict that ignorance inferences in fact project out of various environments. In view of the Strongest Meaning Hypothesis, the negative sentence in (46)a should have the R-free representation in (46)b, without an embedded R. But this fails to derive the diversity inference that is in fact observed. The same remarks extend to the conditional sentence in (46)a.

(46) a. The detective doesn't hope that the award goes to Ann or Bill.
=> the detective thinks it's possible that the award goes to Ann or not, and similarly for Bill
b. not x hope [$A \vee B$]

(47) a. If the detective hopes that the award goes to Ann or Bill, he'll say so.
=> the detective thinks it's possible that the award goes to Ann or not, and similarly for Bill
b. if x hopes [$A \vee B$], C

Importantly, the lexical presupposition of *want* in (32) only derives the result that, for the attitude holder, the entire clause *[A ∨B]* is possibly true and possibly false. The latter requirement (*[A ∨B]* is possibly false) entails that for the attitude holder *A* is possible false and *B* is possibly false. But the former requirement (*[A ∨B]* is possibly true) doesn't entail that for the attitude holder A is possibly true.[9]

When it comes to *[A &B]* embedded under *want*, Blumberg and Goldstein predict once again an R-free representation under negation, as in (38)b, but this fails to account for intuitive ignorance inferences obtained in (48)a. The lexical presupposition in (32) derives part of the result, namely that for the attitude holder *A* is possibly true and *B* is possibly true. But this lexical presupposition does not on its own derive the inference that for the attitude holder *A* is possibly false and *B* is possibly false (e.g. the requirement that the entire conjunction is possibly false could be entirely due to *B*, not to *A*). While one could in principle posit that the conjunction somehow has intermediate scope, yielding a representation equivalent to (48)c, this mere possibility would not explain why the ignorance inference appears to be non-optional.

(48) a. Mary doesn't want Ann to bring apple pie and Bill to bring blueberry pie.
=> Mary thinks it's possible that Ann will or won't bring apple pie, and that Bill will or won't bring blueberry pie
b. not x want [A and B]
c. not [x want A] and not [x want B]

Importantly, this diversity inference couldn't be derived by inserting R despite the Strongest Meaning Hypothesis, for in view of Blumberg and Goldstein's semantics, this would predict that the diversity component is at-issue, and should be affected by the negation, contrary to fact. Whatever inference it is, it seems not to be part of the at-issue contribution of the attitude report.

Further projection problems arise with embedding under existential modals, as in (49)a. In view of the Strongest Meaning Hypothesis, we expect R to appear in embedded position, as in (49)b. But within Blumberg and Goldstein's system (summarized in (17)), failure within the embedded clause translates into falsity (rather than failure) of the immediately embedding clause (in our case: *x want [RA ∨RB]*). As a result, no projection is predicted out of the scope of *maybe*, and the result is arguably too weak, as the predicted inference is that there should be a *possibility* that the detective is ignorant about the embedded disjuncts.

(49) a. Maybe the detective wants Ann or Bill to be indicted.
=> the detective thinks it's possible that Ann will or will not be indicted, and similarly for Bill
b. Maybe x want [RA ∨ RB]

5 Ignorance Inferences without R I: the case of *Believe*

Having laid out the problems with Blumberg and Goldstein's analysis of ignorance inferences, we will now develop an alternative account without R or other syntactic operators. We provide a detailed analysis of disjunctions embedded under *believe* in this section, and then extend it to embedded disjunctions under *hope* in the next section.

5.1 General idea

Our account relies on a similar intuition to that guiding Blumberg and Goldstein's account. First, we propose a generalization of Stalnaker's proposal that "a speaker should not assert what he presupposes to be true, or what he presupposes to be false" (Stalnaker 1978). This non-triviality condition can be generalized to local contexts (this, in turn, can involve dynamic or non-dynamic derivations of local contexts, e.g. Heim 1983 or Schlenker 2009). This leads to the condition in (50).

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[9] Note that we do not discuss *hope* in this connection because the lexical entry in (30) only has a 'possibly false' presupposition, and for embedded *[A ∨B]*, this entails that the agent must take *A* to be possibly false and *B* to be possibly false.

(50) **Stalnaker's non-triviality condition**
If relative to a context set C an expression *F* (of a type that 'ends in t') has a local context c', then conditions a. and b. should both be satisfied:
a. c' $\not\models$ F
b. c' $\not\models$ not F

In several cases, however, the condition in (50) is too weak, for two reasons. First, it is a constraint on the common ground and not on the speaker's beliefs: since the common ground is larger than the speaker's belief set, the constraint is more easily and sometimes too easily satisfied. Second, the local context of an embedded expression, for instance the local context of the disjunction in *x believes [A or B]*, will end up encoding uncertainty about the beliefs that the speech act participants attribute to x. Because the form of (50) is that the local context c' should be 'large enough' that certain entailments shouldn't hold, and because c' will in general be fairly weak, the condition will sometimes be too easy to satisfy.

We propose that in certain circumstances (whose precise nature we will leave open), the conditions in (50) may be strengthened. Our starting point is that the inferences are formally anti-presuppositions: they require that local contexts should *fail* to entail certain things. But it was argued by Chemla 2008 that anti-presuppositions sometimes need to be strengthened, and we will follow a similar guiding principle (though not the details of Chemla's implementation) to obtain a strenghtening of the conditions in (50).

Chemla's (2008) point of departure is Maximize Presupposition, a principle posited that explains why an expression such as *believe* gives rise to an inference that its complement is false. According to this principle, *believe* may only be used if its presuppositional alternative *know* is inapplicable because its presupposition is not met (e.g. Sauerland 2003, 2008; Percus 2006; Singh 2011; Schlenker 2012; Spector and Sudo 2017; Anvari 2018). The net effect of this principle is that a felicitous use of *believe* requires that the common ground does not entail that its complement proposition is true. In short, Maximize Presupposition generates anti-presuppositions (like the Stalnaker non-triviality conditions seen above). But as Chemla notes, this inference is often insufficiently strong, as illustrated in (51).

(51) John believes that I have a sister.
a. Alternative: John knows that I have a sister.
b. Actual inference: The speaker does not have a sister.
c. Predicted inference: It is not common belief that the speaker has a sister.
(Chemla 2008)

Chemla proposes a purely reasoning-based mechanism of strengthening: by combining (51)c with independently plausible principles of epistemic logic, one gets in some desirable cases the stronger inference in (51)b.

Following the spirit but not the details of Chemla's analysis, we will assume that under some conditions the anti-presuppositions in (50) can be strengthened as well. For the embedded disjunct *A* in *x believes [A or B],* the unstrengthened conditions will look like (52)a, with the form: *not [∀w: Cw][∀w': …] …* . The strengthened conditions, by contrast, will take the form *[∀w: Cw] not [∀w': …] …*, as in (52)b (we will shortly discuss the assumptions needed to obtain this strengthening).

(52) **Non-triviality condition for *A* in *x believes [A or B]* relative to a global context set C**
a. Unstrengthened non-triviality condition
not [∀w: Cw][∀w': Dox_x(w)] **A**(w') = 1
not [∀w: Cw][∀w': Dox_x(w)] **A**(w') = 0

b. Strengthened non-triviality condition
[∀w: Cw] not [∀w': Dox_x(w)] **A**(w') = 1
[∀w: Cw] not [∀w': Dox_x(w)] **A**(w') = 0

To obtain the intrusion of ignorance inferences observed by Blumberg and Goldstein, another pragmatic effect might need to come into play. Consider a negative sentence such as *not x believes [A* *or B]*. It will give rise to the same anti-presupposition as in (52)a, and we can assume that in some cases these will also be strengthened to (52)b, where C is now the local context of the clause embedded under *not*, namely *x believes [A or B]*. The boxed parts of (52)b are in effect presuppositions—in this case relative to the local context C. While this local context is identical to the global context, because it is that of a constituent appearing under negation, it could in principle give rise to local accommodation. This, in turn, could explain the intrusion of ignorance conditions in truth conditions, as noted by Blumberg and Goldstein for more complex examples such as (41), whose simplified Logical Form appears in (53).

(53) [Exactly two detectives] $\lambda x\ t_x$ believe [A or B]

At this point, it must be recalled that Chatain and Schlenker 2023a argued on the basis of ellipsis-based data that local accommodation should *not* be implemented by way of an operator, precisely because ellipsis does not give rise to (non-functional) parallelism between the elided expression and its antecedent. If we follow their analysis, we do not expect that local accommodation applied in cases such as (53) should give rise to (non-functional) parallelism constraints under ellipsis.

As an alternative, one might eschew local accommodation entirely, and take the impact of the presupposition on the truth conditions to be due to the lexical semantics of *exactly two*, as predicted by the lexical entry in (34). On that alternative as well, no operator will be needed; the desired effects follow from the pragmatic strengthening of an anti-presupposition into a presupposition, combined with the lexical entry of a numeral.

We will turn to a more detailed implementation of the analysis based on local accommodation.[10]

5.2 Local context computation

We will start with the schematic example in (54). The first order of business is to compute the local context **c'** of the embedded clause, which we write as a superscript on the embedded clause (as before, we write **c'** for the value of the expression *c'*).

(54) x believes $^{c'}$[A or B]

We will follow the details of the account in Schlenker 2009. To ensure that the value of *c'* is a proposition that can vary with the world of utterance (as the attitude holder might hold different beliefs in different worlds), Schlenker 2009 assumes a bidimensional (Kaplanian) framework with two world-like parameters, one, w*, corresponding to the context, and the other, w, corresponding to the world of evaluation. If $Dox_x(w^*)$ is the set of x's doxastic alternatives in w*, the local context of the embedded clause is given in (55). It is a function from contexts w* to propositions of the form λw <u>w* ∈ C</u> and $\boxed{w \in Dox_x(w^*)}$. The underlined condition guarantees that the proposition will be non-empty only if w* is indeed in the global context set. The boxed condition states that when this is the case, **c'**(w*) is just the set of x's doxastic alternatives in w*.

(55) $\mathbf{c'} = \lambda w^*\ \lambda w\ [w^* \in C \text{ and } w \in Dox_x(w^*)]$

5.3 Stalnakerian non-redundancy conditions

With this result, we can already state a Stalnakerian non-triviality condition on *A*. We'll write **A**(w*)(w) the value of A evaluated relative to context w* and world of evaluation w. Stalnaker's non-triviality conditions appear in (56), and can be rewritten as in (57).

(56) a. $\mathbf{c'} \not\models A$
b. $\mathbf{c'} \not\models \text{not } A$

[10] The accommodation-free alternative is more difficult to implement within the analysis of local contexts we adopt because the latter does not predict the existential projection needed for the proposed lexical entry of *exactly two* (rather, it predicts universal projection).

(57) a. not $\forall w^* \forall w$ [$w^* \in C$ and $w \in Dox_x(w^*)$ => $\mathbf{A}(w^*)(w) = 1$],
i.e. not [$\forall w^*$: $w^* \in C$] [$\forall w$: $w \in Dox_x(w^*)$] $\mathbf{A}(w^*)(w) = 1$

b. not $\forall w^* \forall w$ [$w^* \in C$ and $w \in Dox_x(w^*)$ => $\mathbf{A}(w^*)(w) = 0$],
i.e. not [$\forall w^*$: $w^* \in C$] [$\forall w$: $w \in Dox_x(w^*)$] $\mathbf{A}(w^*)(w) = 0$

Following the theories of Beaver 2001 and Schlenker 2009, the local context **c**" of the second embedded disjunct is obtained by intersecting the local context of the entire disjunction with the negation of the first disjunct. This gives the value in (58), and the non-triviality conditions in (59).

(58) **c" =** $\lambda w^* \lambda w$ [$w^* \in C$ and $w \in Dox_x(w^*)$ and $\mathbf{A}(w^*)(w) = 0$]

(59) a. not [$\forall w^*$: $w^* \in C$] [$\forall w$: $w \in Dox_x(w^*)$ and $\mathbf{A}(w^*)(w) = 0$] $\mathbf{B}(w^*)(w) = 1$
b. not [$\forall w^*$: $w^* \in C$] [$\forall w$: $w \in Dox_x(w^*)$ and $\mathbf{A}(w^*)(w) = 0$] $\mathbf{B}(w^*)(w) = 0$

These non-triviality conditions are rather weak. This avoids a problem that arose for Blumberg and Goldstein. As we saw, representations like *x believes RA or RB* were contradictory, since *B* was entailed by *not A* in x's doxastic alternatives given the assertion. This structure thus had to be ruled out independently. While very similar to non-redundancy conditions, our non-triviality conditions are stated with respect to the common ground. They can be satisfied even when the speaker believes that *B* is entailed by *not A* in x's doxastic alternatives, as long as the common ground, a larger set than the speaker's belief set, contains a world in which this isn't the case. To put it differently, the contradiction problem does not arise as long as it is not *presupposed* that *x believes A or B*.

Another advantage afforded by the weakness of our non-triviality conditions is that it accounts for the deviance of sentences such as (60)a. Within Blumberg and Goldstein's framework, the candidate structures in (60)a',a" are ruled out by Contradiction Avoidance. Thus the only option for such a sentence is the R-free representation in (60)a, but no deviance is predicted for it. By contrast, we predict that our non-triviality conditions are violated in (60)a, since any common ground fails to satisfy (59)b when A = B.

(60) a. #x believes [A or A]
a'. x believes [RA or A]
a". x believes [RA or RA]
b. #My mother believes that I am at home or at home.

5.4 Strengthening

Our non-triviality conditions are the first step towards the desired ignorance inferences. The next step consists in a strengthening of these non-triviality conditions. Without deriving it from first principles (unlike Chemla 2008), we model it as a form of homogeneity with respect to propositions. Our notion of homogeneity is defined in (61).

(61) **Homogeneity with respect to a proposition**

A context set C is homogeneous with respect to a proposition p just in case:

[$\forall w^*$: $w^* \in C$] $p(w^*) = 1$ if and only if [$\exists w^*$: $w^* \in C$] $p(w^*) = 1$
[$\forall w^*$: $w^* \in C$] $p(w^*) = 0$ if and only if [$\exists w^*$: $w^* \in C$] $p(w^*) = 0$
(…from which it follows that
[$\forall w^*$: $w^* \in C$] $p(w^*) = \#$ if and only if [$\exists w^*$: $w^* \in C$] $p(w^*) = \#$)

Note: Since the equivalences have the form
[$\forall w^*$: $w^* \in C$] F if and only if [$\exists w^*$: $w^* \in C$] F,
we also have
not [$\forall w^*$: $w^* \in C$] F iff not [$\exists w^*$: $w^* \in C$] F, and
[$\exists w^*$: $w^* \in C$] not F iff [$\exists w^*$: $w^* \in C$] not F
and thus in particular

$[\exists w^*: w^* \in C]\ p(w^*) \neq 1$ if and only if $[\forall w^*: w^* \in C]\ p(w^*) \neq 1$
$[\exists w^*: w^* \in C]\ p(w^*) \neq 0$ if and only if $[\forall w^*: w^* \in C]\ p(w^*) \neq 0$

Applied to the anti-presupposition of Chemla's example in (51), Homogeneity yields the desired strengthening, as can be seen in (62).

(62) a. John believes that I have a sister.
Notation: we write as **sister** the proposition that the speaker has a sister.
b. Result of Maximize Presupposition
not $[\forall w^*: w^* \in C]$ **sister**$(w^*) = 1$, hence
$[\exists w^*: w^* \in C]$ **sister**$(w^*) \neq 1$
b. Homogeneity applied to **sister**
$[\forall w^*: w^* \in C]$ **sister**$(w^*) \neq 1$
c. Result: It is common belief that the speaker does not have a sister.

In the case of interest, *x believes [A or B]*, there are four anti-presuppositions to be considered, listed in (63). Each could in principle be strengthened using homogeneity, with the results in (64).

(63) Stalnakerian non-triviality conditions for *x believes [A or B]*
a. not $[\forall w^*: w^* \in C]\ [\forall w: w \in Dox_{\mathbf{x}}(w^*)]\ \mathbf{A}(w^*)(w) = 1$
b. not $[\forall w^*: w^* \in C]\ [\forall w: w \in Dox_{\mathbf{x}}(w^*)]\ \mathbf{A}(w^*)(w) = 0$
a'. not $[\forall w^*: w^* \in C]\ [\forall w: w \in Dox_{\mathbf{x}}(w^*)$ and $\mathbf{A}(w^*)(w) = 0]\ \mathbf{B}(w^*)(w) = 1$
b'. not $[\forall w^*: w^* \in C]\ [\forall w: w \in Dox_{\mathbf{x}}(w^*)$ and $\mathbf{A}(w^*)(w) = 0]\ \mathbf{B}(w^*)(w) = 0$

(64) Candidate strengthenings for (63), using Homogeneity
a. $[\forall w^*: w^* \in C]$ not $[\forall w: w \in Dox_{\mathbf{x}}(w^*)]\ \mathbf{A}(w^*)(w) = 1$
b. $[\forall w^*: w^* \in C]$ not $[\forall w: w \in Dox_{\mathbf{x}}(w^*)]\ \mathbf{A}(w^*)(w) = 0$
a'. $[\forall w^*: w^* \in C]$ not $[\forall w: w \in Dox_{\mathbf{x}}(w^*)$ and $\mathbf{A}(w^*)(w) = 0]\ \mathbf{B}(w^*)(w) = 1$
b'. $[\forall w^*: w^* \in C]$ not $[\forall w: w \in Dox_{\mathbf{x}}(w^*)$ and $\mathbf{A}(w^*)(w) = 0]\ \mathbf{B}(w^*)(w) = 0$

The conditions in (64)a,b amount to an ignorance inference about **A**. Since the speaker's belief set is a subset of the context set C, these conditions entail Blumberg and Goldstein's non-redundancy conditions. As is the case in Blumberg and Goldstein's analysis, the strengthened inferences in (64)a',b' would contradict the speaker's statement. Like Blumberg and Goldstein, we can prevent this strengthening by invoking Contradiction Avoidance. Importantly, Contradiction Avoidance only prevents *strengthening* of the non-triviality conditions; the non-triviality conditions themselves arise regardless. This means that the infelicity of *x believes [A or A]* discussed in the previous section is still predicted, as it originates in a violation of the *unstrengthened* non-triviality conditions.[11]

5.5 Accommodation

Blumberg and Goldstein's conditions are at-issue. For unembedded clauses, applying global accommodation to the strengthened inferences (formally, presuppositions) yields the same result. If we wish to explain the embeddability of the ignorance inferences they observe in (65)a, we can simply apply local accommodation, and rely on operator-free analyses of this phenomenon, for instance that developed in Chatain and Schlenker 2023a. Alternatively, one may be able to do everything without local accommodation by using the projection rules for *exactly two* proposed in (34).

[11] A reviewer notes that, by the same token, "the detective isn't certain that Ann or Bill committed the crime" should trigger an inference that the detective considers Ann a possible suspect and Bill too. The only reason why such an inference may not be derived would be if it contradicts the assertion and what is known. But in the present case, it is coherent to assume that the detective thinks it is possible that either of them did it, while not being certain. It is unclear whether this inference exists under some reading. Figuring out whether this inference exists is something we leave to future research.

(65) a. Exactly two detectives believe that Ann or Bill committed the crime.
b. [Exactly two detectives] $\lambda x\ t_x$ believe [A or B]

6 Ignorance Inferences without R II: the case of *Hope*

The same analysis can be applied to embedding under *hope*, with some tweaks.

6.1 Local context and Stalnakerian non-redundancy conditions

In our analysis of *believe [A or B]*, we relied on the theory of local contexts proposed in Schlenker 2009 to derive the local context of each disjunct. But Schlenker 2009 does not extend this result to other attitude verbs than *believe* like *hope*. Blumberg and Goldstein postulates that the local context for such verbs would be the same as with *believe*, namely (66).

(66) $\mathbf{c'} = \lambda w^*\ \lambda w\ [w^* \in C \text{ and } w \in Dox_{\mathbf{x}}(w^*)]$

This is a reasonable assumption from the perspective of presupposition projection: attitudes like *hope* presuppose that the agent believes the presupposition of the embedded clause.[12] We adopt this assumption without deriving it (but see Blumberg and Goldstein, to appear).

(67) a. x hopes [A or B]
b. The detective hopes Ann or Bill committed the crime.

With this assumption, the embedded local contexts are the same in (67)a as in the case of *believe*, and we derive the same non-triviality conditions as with *believe*, stated in (63).

6.2 Strengthening and local accommodation

By contrast with *believe*, with *hope* all non-triviality conditions may be strengthened by homogeneity reasoning without contradicting what is asserted. This yields the inferences listed in (68). This result directly mirrors Blumberg and Goldstein's observation that with any attitude other than *believe*, the insertion of *R* on each disjunct of embedded disjunction is not contradictory.

(68) Candidate strengthenings for (63) using Homogeneity
a. $[\forall w^*\!: w^* \in C]$ not $[\forall w\!: w \in Dox_{\mathbf{x}}(w^*)]\ \mathbf{A}(w^*)(w) = 1$
b. $[\forall w^*\!: w^* \in C]$ not $[\forall w\!: w \in Dox_{\mathbf{x}}(w^*)]\ \mathbf{A}(w^*)(w) = 0$

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[12] We leave aside the notorious fact that a factive presupposition is often obtained on top of the doxastic presupposition, as seen in the second inference in (i)a. One can view this as an instance of the Proviso Problem, whereby unsupported presuppositions give rise to stronger accommodation than is predicted by current theories of local contexts, including Heim 1983 and Schlenker 2009 (see for instance Geurts 1996, 1999, Lassiter 2012, Mandelkern 2016). One argument for this analysis is that when the doxastic presupposition is explicitly justified, as in(i)b, the factive presupposition disappears. For instance, the following examples seem (to our ear) to yield the following inferences:

(i) a. <> Does your mother hope that you'll continue to be the most attractive person on earth?
=> your mother thinks you currently are the most attractive person on earth
=>? you currently are the most attractive person on earth

b. <>Your mother is certain that you are the most attractive person on earth, but does she hope you'll continue to be into old age?
≠> your mother thinks you currently are the most attractive person on earth
≠> you currently are the most attractive person on earth

a'. $[\forall w^*: w^* \in C]$ not $[\forall w: w \in Dox_x(w^*)$ and $\mathbf{A}(w^*)(w) = 0]$ $\mathbf{B}(w^*)(w) = 1$
b'. $[\forall w^*: w^* \in C]$ not $[\forall w: w \in Dox_x(w^*)$ and $\mathbf{A}(w^*)(w) = 0]$ $\mathbf{B}(w^*)(w) = 0$

Taken together, the strengthened inferences imply two things: first, that the agent is ignorant about *A* ((68)a and b together), second, that she is ignorant about *B* ((68)a' and b' together). These are the desired ignorance inferences.[13] Just as with *believe,* intrusion of ignorance inferences may be predicted in various ways: either by local accommodation or by projection out of *exactly two*.[14]

7 Conclusion

We conclude that the non-redundancy operator R is not syntactically real. Ellipsis-based tests suggest that, as it stands the operator is undesirable. Furthermore, its effects are better analyzed by way of operator-free pragmatic processes, and specifically by two independently motivated mechanisms. Diversity inferences in the immediate scope of some attitude verbs are due to their lexical presuppositions, as already posited by Heim 1982. Ignorance inferences triggered by embedded disjunctions and conjunctions have a different source: they arise from Stalnakerian conditions that prohibit an expression from being trivial relative to its local context. These inferences are technically anti-presuppositions, but like the latter more generally, they can sometimes be strengthened into presuppositions. And these can sometimes enter in at-issue truth conditions, whether by way of accommodation (implemented operator-free) or because of the lexical semantics of numeral quantifiers.

We leave several important questions for future research. First, our account of ignorance inferences triggered by conjunctions and disjunctions crucially relies on mechanisms to strengthen anti-presuppositions into presuppositions. We have argued that there is independent evidence for such strengthening mechanisms, but we have not sought to derive their precise form from first principles; this has yet to be done.

Second, our ellipsis arguments against the syntactic reality of R may not extend to refinements of this operator. As argued by Chatain and Schlenker 2023b, non-parallelism in ellipsis for the Exh pragmatic operator may be predicted if one is willing to make two assumptions: (i) that the operator must be present in every position, (ii) the operator's semantics is dependent on contextual parameters in such a way that, for some value of the parameter, it is vacuous but not others. With these assumptions, a "sloppy" reading of the operator is possible, which is indistinguishable from the non-parallelism we observe. We leave it to future research to determine whether similar assumptions could be adopted in this context, but we preliminarily note that the assumptions (i) and (ii) make the syntactic operator nearly indistinguishable from a semantic or pragmatic process.

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[13] We also derive diversity inferences relative to the entire embedded clause, stated in (i). But these are redundant with the diversity inferences that are triggered lexically by *hope* in the account we developed in Section 3.2.

(i) a. $[\forall w^*: w^* \in C]$ not $[\forall w: w \in Dox_x(w^*)]$ $(\mathbf{A}(w^*)(w) = 1$ or $\mathbf{B}(w^*)(w) = 1)$
b. $[\forall w^*: w^* \in C]$ not $[\forall w: w \in Dox_x(w^*)]$ $(\mathbf{A}(w^*)(w) = 0$ and $\mathbf{B}(w^*)(w) = 0)$

In greater detail: from (68)b', it follows that for every w* in C, $[\exists w: w \in Dox_x(w^*)$ and $\mathbf{A}(w^*)(w) = 0]$ $\mathbf{B}(w^*)(w) \neq 0$, which establishes both (i)a and (i)b.

[14] More generally, our analysis predicts that other anti-presuppositions could intrude under quantifiers. We are unclear about the data. The anti-presupposition of *believe p* (e.g. *p* is false) appears *prima facie* embeddable, as in (i):

(i) Exactly two of our applicants believe they will get the job. The third one knows it.

(i) is judged to be felicitous by our consultants. But this 'anti-presupposition' of *believe* can plausibly be reanalyzed as a regular implicature, by competition with a bivalent (i.e. non-presuppositional) version of *know*. If so, (i) may only support the embeddability of implicatures, not necessarily of anti-presuppositions. More work is needed on this topic.

Third, Blumberg and Goldstein motivated their account on the basis of yet another set of data, pertaining to Free Choice readings. The short of it is that the ellipsis test can serve to refute their account, but this can be done using observationchs that are already in the literature. Because there is such a large body of work on Free Choice readings, we leave this issue for future research, but we summarize in the Appendix extant and new objections against an account based on R.

Appendix.
Free Choice Without R

In this appendix, we briefly discuss Blumberg and Goldstein's account of some Free Choice effects using their operator R. Without giving a full account, we argue (i) that the R-based account is in error, and (ii) that its positive features can be retained, operator-free, on the basis of Stalnakerian conditions of non-triviality. (We emphatically do not seek to develop a full account of Free Choice inferences, a topic that goes beyond the present piece.)

❑ *Blumberg and Goldstein's data and analysis*

Blumberg and Goldstein 2021b apply their framework to Free Choice inferences, illustrated in (69)a, with the representation in (69)b.

(69) a. Mary may have apples or bananas.
=> Mary may have apples
=> Mary may have bananas
b. may [RA ∨ RB]

The local context c' of the embedded clause is the set of deontically accessible possible worlds, and *RA* guarantees that <u>some worlds in c' satisfy A</u> and others don't. The local context of the second disjunct is c' ∩ **(not A)**, and this further guarantees that within c' <u>some</u> (non-A) <u>worlds satisfy B</u> and others don't. The underlined components account for the free choice inference.

Blumberg and Goldstein further propose that the insertion of R under universal modals such as *is required* or *must* is responsible for a diversity inference, as illustrated in (70)a.

(70) a. Mary is required to read Ulysses or Madame Bovary.
=> Mary may read Ulysses and Mary may read Madame Bovary
b. must [RA ∨ B]
c. #must [RA ∨ RB]

The diversity inference can be derived by way of the representation in (70)b, where once again the local context c' of the embedded clause is the set of deontically accessible worlds, while the local context of the second disjunct is c' ∩ **(not A)**. Due to the universal nature of the modal, the representation in (70)c is inconsistent, as it places contradictory demands on the set of deontically accessible worlds: all should satisfy A or B, but in addition some not-A worlds should satisfy not-B (to fulfill the non-triviality requirement of B relative to its local context). No such problem arises with the representation in (70)b.

❑ *Problems with ellipsis*

As was the case for attitude verbs, Blumberg and Goldstein endow modals with a bivalent semantics, with the result that non-redundancy conditions within their scope end up affecting at-issue truth conditions. In elided clauses, one may have no choice but to copy an R-full antecedent. This situation arises in (71)a, which should have the representation in (71)b. The latter predicts that the elided clause negates a clause enriched with non-redundancy conditions. By negating this strong meaning, the overall result is too weak: it fails to license the inference that Bill can't have apples and Bill can't have bananas.

(71) a. Mary can have apples or bananas. Bill can't.
=> Mary may have apples and Mary may have bananas
=> Bill can't have apples and Bill can't have bananas
b. may M λx [RAx ∨ RBx]. not may B λx [RAx ∨ RBx]

In fact, the role of ellipsis in constraining theories of Free Choice was discussed in Bar-Lev and Fox 2017. Starting from an account of Free Choice in terms of double exhaustification (Fox 2007), they noticed that the ellipsis facts in (72)a can be captured if the ellipsis site is small enough, as in (72)b.

(We follow Bar-Lev and Fox in notating the exhaustivity operator as *Exh* rather than as *O*, as was the case at the beginning of this piece.)

(72) a. Mary is allowed to eat ice cream or cake, and John isn't <allowed to eat ice cream or cake>.
=> Mary is allowed to eat ice cream and allowed to eat cake, and
=> John isn't allowed to eat ice cream and he isn't allowed to eat cake.
(Bar-Lev and Fox 2017, handout)

b. Exh Exh Mary λx allowed [Ax ∨ Bx]. John isn't <λx allowed [Ax ∨ Bx]>.

But Bar-Lev and Fox also noticed that the quantified case in (73)a requires double exhaustification under the universal quantifier, yielding the expectation that a Free Choice reading should be obtained under the scope of the negative quantifier with ellipsis, as in (73)b.

(73) a. Every girl is allowed to eat ice cream or cake on her birthday. Interestingly, no boy is allowed to eat ice cream or cake on his birthday.
=> every girl is allowed to eat ice cream and allowed to eat cake on her birthday, and
=> no boy is allowed to eat ice cream and (likewise) no boy is allowed to eat cake on his birthday.
b. [every girl] λx Exh Exh allowed [Ax ∨ Bx]. [no boy] λx Exh Exh allowed [Ax ∨ Bx].
c. Exh^{II} [every girl] λx allowed [Ax ∨ Bx]. [no boy] λx allowed [Ax ∨ Bx].

But the representation in (73)b predicts an overly weak reading, one that doesn't derive the observed inferences. Bar-Lev and Fox took this to be one piece of evidence for replacing the standard exhaustivity operator (based on 'innocent exclusion') with a new one, based on 'innocent inclusion'. The crucial observation was that the latter could be given matrix scope and still yield the desired reading in the first (universal) sentence of (73)a, as displayed in (73)c (where Exh^{II} is the newly defined exhaustivity operator). This configuration allowed ellipsis to target an operator-free constituent, as was desired in view of the intuitive truth conditions.

Importantly, from the perspective of Blumberg and Goldstein's theory, *both* (72)a and (73)a present a challenge, as in both cases the operator R must be embedded within the disjunctions and must thus be copied by ellipsis, contrary to fact.

Turning to *must*, Blumberg and Goldstein's theory *on its own* makes incorrect predictions for negated sentences, but this is for irrelevant reasons, as independently motivated assumptions about implicatures can address the problem. Specifically, the Strongest Meaning Hypothesis implies that the representation of (74)a should be (74)b, which fails to derive the appropriate diversity inferences. But the latter can be derived as implicatures in view of the alternatives in (74)c: by negating the alternatives *Mary may not study Greek, Mary may not study Latin*, the desired result is obtained.

(74) a. Mary is not required to study Greek or Latin.
=> Mary can study Greek
=> Mary can study Latin
b. not must M λx [Gx ∨ Lx]
c. Alternatives: {not may M λx [Gx ∨ Lx], not may M λx Gx, not may M λx Lx, …}
=> may M λx Gx
=> may M λx Lx

With implicatures at our disposal, the diversity inferences in (70)a, repeated as (75)a, can be analyzed without R: by the assertion, all deontically accessible worlds satisfy *U* or *B*; by the implicatures in (75)b, not all satisfy *U* and not all satisfy *B*, from which it follows that some satisfy *U* and *not B* and some satisfy *B* and *not U*.

(75) a. Mary is required to read Ulysses or Madame Bovary.
=> Mary may read Ulysses and Mary may read Madame Bovary
b. Alternatives: {must M λx Ux, must M λx Bx, …}
=> not must M λx Ux
=> not must M λx Bx

hence with the assertion
=> may M λx Ux, may M λx not Ux, may M λx Bx, may M λx not Bx

While in this case there may be a choice between R-insertion and implicatures, with ellipsis the R-based account encounters difficulties. The problem can be seen in (76)a, where the antecedent cannot contain an embedded R due to the Strongest Meaning Hypothesis. Nonetheless, the elided clause does give rise to this diversity inference, despite the fact that the copied clause is R-free. By contrast, an implicature-based analysis derives the correct result, for instance by postulating the presence of an exhaustivity operator in the unelided part of the positive clause, as displayed in (76)c.

(76) a. Mary is not required to study Greek or Latin, but Bill is.
=> Bill can study Greek, Bill can study Latin
b. not must M $\lambda x\, [Gx \vee Lx]$. must B $\langle \lambda x\, [Gx \vee Lx] \rangle$
c. not must M $\lambda x\, [Gx \vee Lx]$. **Exh** must B $\langle \lambda x\, [Gx \vee Lx] \rangle$

❑ *Stalnakerian conditions*

Since Bar-Lev and Fox's (2017) implicature-based theory is designed to address problems raised by ellipsis, we do not need to offer an alternative account. Suffice it to say that some of Blumberg and Goldstein's results could, if one wanted, be derived by positing Stalnakerian non-triviality conditions (anti-presuppositions) which could be strengthened into presuppositions and then locally accommodated under some pragmatic conditions.

Let us focus on Free Choice under *may*. The account is initially similar to that given for *x believes [A or B]* in Section 5. The target representation is in (77), with an embedded local context *c*'. We will assume that its value is obtained in the same way as the local context of the embedded clause under *believe*, but with a set of deontic alternatives (written as *Deont*) replacing the set of doxastic alternatives, as shown in (78).

(77) may $^{c'}$[A or B]

(78) **c' =** $\lambda w^*\, \lambda w\, [w^* \in C \text{ and } w \in Deon(w^*)]$

The Stalnakerian non-triviality conditions appear in (79) and give rise to the candidate strengthenings in (80). All are consistent with the assertion, and (80)b and (80)a' derive the Free Choice inference.[15] When local accommodation is applied, we obtain an at-issue effect, and hence something very close to Blumberg and Goldstein's conditions.

(79) Stalnakerian non-triviality conditions for *May [A or B]*
a. not $[\forall w^*\!: w^* \in C]\ \underline{[\forall w\!: w \in Deon(w^*)]\ \mathbf{A}(w^*)(w) = 1}$
b. not $[\forall w^*\!: w^* \in C]\ \underline{[\forall w\!: w \in Deon(w^*)]\ \mathbf{A}(w^*)(w) = 0}$
a'. not $[\forall w^*\!: w^* \in C]\ \underline{[\forall w\!: w \in Deon(w^*) \text{ and } \mathbf{A}(w^*)(w) = 0]\ \mathbf{B}(w^*)(w) = 1}$
b'. not $[\forall w^*\!: w^* \in C]\ \underline{[\forall w\!: w \in Deon(w^*) \text{ and } \mathbf{A}(w^*)(w) = 0]\ \mathbf{B}(w^*)(w) = 0}$

(80) Candidate strengthenings using Homogeneity relative to the underlined propositions of 26
a. $[\forall w^*\!: w^* \in C]$ not $[\forall w\!: w \in Deon(w^*)]\ \mathbf{A}(w^*)(w) = 1$
hence: May not A
b. $[\forall w^*\!: w^* \in C]$ not $[\forall w\!: w \in Deon(w^*)]\ \mathbf{A}(w^*)(w) = 0$
hence: May A
a'. $[\forall w^*\!: w^* \in C]$ not $[\forall w\!: w \in Deon(w^*) \text{ and } \mathbf{A}(w^*)(w) = 0]\ \mathbf{B}(w^*)(w) = 1$
hence: May (not A and not B)

[15] In the case of *believe*, which has the force of universal quantification of worlds, the counterpart of (80)a' was inconsistent with the assertive component and couldn't be effected. But *may* has existential rather than universal force, and the problem does not arise.

b'. $[\forall w^*: w^* \in C]$ not $[\forall w: w \in Deon(w^*)$ and $\mathbf{A}(w^*)(w) = 0]$ $\mathbf{B}(w^*)(w) = 0$
hence: May (not A and B)

We caution that these strengthenings are not unproblematic, for two related reasons. First, the presuppositions in (80) are too strong: *may [A or B]* intuitively yields the inference that the *speaker* believes that *may A* and also *may B*, but not that these modal propositions are presupposed. Second, these presuppositions make the at-issue component of *may [A or B]* redundant. This might conceivably explain why the presupposition is automatically accommodated (thus solving the first problem). But an alternative is that these strengthenings do not arise in the first place, and that an implicature is responsible for the Free Choice effect.

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