Cumulativity from the perspective of homogeneity

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Abstract

Cumulative readings of quantifiers like *every* (Champollion, 2010; Haslinger and Schmitt, 2018; Kratzer, 2003; Schein, 1993) seem to defy traditional rules of composition. This paper offers a new analysis of such readings which aims to preserve both the semantics of the quantifiers and traditional compositional rules. It starts with the observation that the truth-conditions of the negation of these sentences are unproblematic: a single stipulation about the meaning of the verb yields appropriate truth-conditions for the examples considered. Taking this as a starting point, this paper then extends the analysis to positive sentences using mechanisms for strengthening akin to those proposed by Bar-Lev (2018b) in the context of homogeneity. The resulting analysis captures not only cumulative readings of *every* and other quantifiers but also subject/object asymmetries regarding the presence of these readings (Haslinger and Schmitt, 2018; Kratzer, 2003).

1 The problem of cumulative readings of quantifiers

1.1 Ordinary cumulativity

When two or more plural referential expressions are arguments of the same verb, they often give rise to the so-called cumulative reading (Jackendoff, 1972; Scha, 1984; Sternefeld, 1998). In (1), the cumulative reading asserts that the cooks and the oysters were involved in some opening but does not specifically say which of the cooks opened which of the oysters. Assuming that an oyster can only be opened by one cook[1], (1b) is the paraphrase of the truth-conditions:

(1) a. The 10 cooks opened the 15 oysters.
    b. **Truth-conditions:**
       Every one of the 10 cooks opened an oyster.
       Every one of the 15 oysters was opened by a cook.

Cumulative sentences of the form in (1a), with two referential arguments and a transitive verb, will be referred to as *ordinary cumulative sentences*.

As noted by Roberts (1987), the cumulative truth-conditions of these sentences do not raise issues for the semanticist. A simple analysis would treat (1a) as a plain predication[2], as in (2a).

[1] This paraphrase, found in Scha (1984) but made prominent in Sternefeld (1998) isn't adequate when collective actions are possible, for instance if more than one cook collaborate to open one oyster. For most of the article, I will set aside collective action. A partial solution is given in section 5.2.

Under that view, the cumulative truth-conditions observed in (1b) are part of the meaning of the word *open*. Specifically, one can assume that the denotation of *open* obeys the *cumulative stipulation* given in (2b). This analysis of the cumulative truth-conditions is *prima facie* plausible and has been pursued in Roberts (1987); Scha (1984).

(2) a. $[\![\text{opened}]\!](\iota\text{15-oysters})(\iota\text{10-cooks})$

b. **Cumulative lexical stipulation:**
$[\![\text{opened}]\!](X)(Y)$ iff
every one of Y opened one of X
and every one of X was by opened one of Y

1.2 The problem of cumulative readings of *every*

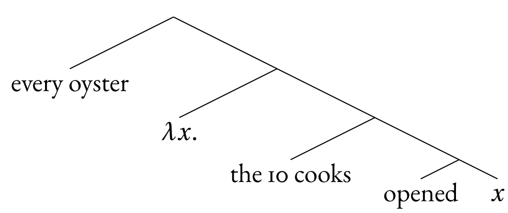
The cumulative stipulation in (2b)[3] generates problematic predictions outside of ordinary cumulative sentences. Consider (3a), where the object argument is replaced with the quantifier *every* (hereafter called cumulative sentences with *every*). This sentence has a cumulative reading, like the original sentence in (1), i.e. the reading in (3b).

(3) a. The 10 cooks opened every oyster.

b. **Truth-conditions:**
Every one of the 10 cooks opened an oyster.
Every oyster was opened by a cook.

The problem is that the cumulative stipulation in (2b), together with very standard assumptions about composition, doesn't derive this cumulative reading. Since *every* is a universal quantifier, one expects (3a) to be paraphrasable as: "*for every oyster x, the cooks opened x*". This is indeed what one derives by applying the compositional rules of e.g. Heim and Kratzer (1998), as is done in (4b). Because of the cumulative lexical stipulation, this paraphrase is in turn equivalent to "*every cook opened every oyster*". This is a possible reading of the sentence but not the cumulative reading of (3b) we are interested in.

(4) a.



b. $[\![(4a)]\!]$ is true
iff $\forall x \in \text{oyster}, \text{open}(x)(\iota\text{cooks})$

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[2] Following Link (1983), I assume that singular and plural DPs both denote elements of D_e (in particular, plural DPs do not denote sets) and that D_e forms a semi-lattice. We assume that singular count DPs denote atoms. $X \prec Y$ is written to mean that the plurality X is contained in or equal to Y. $X + Y$ is written to mean the smallest entity that contains X and Y. We adopt a convention used in Barker (2007) a.o.: lower-case variables (x, y) range over singularities, upper-case variables (X, Y) range over singularities and pluralities alike.

[3] All examples reported in this work are either adaptations of examples from the literature (positive cumulative sentences and non-cumulative sentences) or constructed English sentences checked with four native speakers of English (negative cumulative sentences).

(by compositional rules)
iff $\forall x \in \mathsf{oyster}, \left(\forall y < \iota\mathsf{cooks}, \mathsf{open}(x)(y)\right) \wedge \left(\exists y < \iota\mathsf{cooks}, \mathsf{open}(x)(y)\right)$
(by cumulative denotation of *open*[4])
iff $\forall x \in \mathsf{oyster}, \mathsf{open}(x)(\iota\mathsf{cooks})$
(by simplification)
i.e. "*every cook opened every oyster*"

every is not an exception among quantifiers in giving rise to cumulative readings. The examples in (5-6) are all examples of cumulative readings with various quantifiers.

(5) a. The ten cooks opened between 28 and 35 oysters.
b. **Truth-conditions:**
Between 28 and 35 oysters were opened by some of the ten cooks.
Every one of the ten cooks opened an oyster.

(6) a. The ten cooks opened a prime number of oysters.
b. **Truth-conditions:**
A prime number of oysters were opened by some of the ten cooks.
Every one of the ten cooks opened an oyster.

The examples above are less interesting to the theorist[5]. The quantifiers involved are quantifiers over pluralities and the truth-conditions of these sentences arise naturally from composition and the cumulative meaning postulate of (2b).

There are, however, some suggestive analogues to the case of *every* with plural quantifiers. The case of the non-partitive quantifier *most Ns* is an example. Despite being marked plural, this quantifier is often[6] read as if it ranged over singularities like *every* (Crnič, 2010; Kamp and Reyle, 2013). This is illustrated in (8) (adapted from Crnič (2010)): in both sentences, only a distributive reading is possible.

(8) a. # Most cooks opened the ten oysters.
≈ *a majority of cooks is such that* ***each*** *of them opened the oysters.*
≠ *a majority of cooks is such that they opened the 10 oysters (together).*
b. Most lawyers$_1$ hired a secretary they$_1$ liked.
≈ *each member of a majority of lawyers is such that they hired a secretary they liked.*

[4] One could reply that the lexical stipulation in (2b) only applies when X and Y are plural entities. To dismiss this reply, it suffices to note that the reading received by *The cooks opened the oyster* still requires that all cooks opened the oyster. The formally similar *The cooks opened* x, with x ranging over singularities, should receive that same reading, opening the path to the paradox.

[5] Similar sentences are more interesting. Particularly well-studied is the case of cumulative readings of two modified numerals (Brasoveanu, 2013; Buccola and Spector, 2016; Landman, 2000). Dealing with such examples is outside the scope of this paper.

[6] This isn't always so. As noted by Kamp and Reyle (2013), collective predicates can combine with non-partitive *most* with varying degrees of acceptability. Just as with *every*, we can construct a video-game example to show that *most* retains its distributive semantics even when construed cumulatively:

(7) The ten video-games taught most quarterbacks three new plays.

≠ the members of a majority of lawyers are such that they hired a secretary they all liked.

In spite of this, cumulative readings of *most Ns* are possible when the latter stands in object position, just like *every*.

(9) The ten cooks opened most oysters.
≈ there is majority of oysters such every cook opened one of them and each one of them was opened by a cook

These facts suggest that the problem of cumulative readings in object position is not restricted to *every*. This fact illustrates that a solution to the problem of cumulative readings in object position had better not rely on a particular semantics of *every* but should apply fully generally to other quantifiers as well.

1.3 Outlook and roadmap

The problem of cumulative sentences with *every* is well-known: it was brought to the attention of the semantics literature by Schein (1993). Solutions have been proposed in many other works such as Brasoveanu (2013); Champollion (2010, 2016); Ferreira (2005); Haslinger and Schmitt (2018); Ivlieva (2013); Kratzer (2000); Lasersohn (1990), using various tools such as event semantics, cross-categorial pluralities or different denotation for a quantifier like *every*.

This paper seeks to offer a different resolution of the challenge raised by cumulative readings of *every*. In contrast to previous approaches, I will focus not on the truth-conditions of the positive cumulative sentence in (3a) but on the truth-conditions of its negative counterpart, which are seldom discussed. Importantly, the truth-conditions of negative cumulative sentences with *every* are not simply obtained by complementing the truth-conditions of the positive sentences, an instance of the broader phenomenon of *homogeneity* (Löbner, 2000). The critical observation, as we will see, is that the truth-conditions of the negative sentence can be accounted for without altering the composition or the semantics of quantifiers, changing only our lexical stipulations[7].

While successful on negative sentences, this analysis will miss some of the inferences attested in positive cumulative sentences. To extend the account, I will adopt and adapt Bar-Lev (2018b)'s account of homogeneity and, following him, propose to treat the missing inferences as implicatures. The missing inferences, as we'll see, resemble known cases of Free Choice and distributive implicatures and we can port an existing account of these implicatures to the case of cumulativity.

This approach yields the following theoretical and empirical payoffs: it maintains traditional rules of composition and keeps intact the semantics of quantifiers, including *every*, relying entirely on assumptions about implicature calculation which can be independently grounded. The conservative compositional semantics allows one to straightforwardly extend the approach to cumulative readings of other quantifiers, one of the desiderata of the previous section. In other approaches, cumulative readings of new quantifiers either cannot be integrated or each

[7] This is the opposite of the strategy followed by Haslinger and Schmitt (2020), who start from a strong positive reading and subsequently weaken it.

requires its own set of assumptions. Second, this account does justice to the truth-conditions of negative cumulative sentences, which previous accounts often do not discuss or capture as stated. Finally, this account predicts interesting subject/object asymmetries regarding the availability of the cumulative reading (which are captured by other accounts as well).

The roadmap is as follows: Section 2 presents how the homogeneity properties of cumulative sentences shed light on the problem of cumulative readings ; it proceeds to present the account at bird-eye's view, by analogy with Free Choice and distributive implicatures, extending Bar-Lev (2018b)'s proposal for non-cumulative sentences. Section 3 fleshes out the account formally in terms of recursive exhaustification. Section 4 fleshes out the analysis on several more cases: subject/object asymmetries, cumulative readings of other quantifiers than *every*, including non-partitive *most*, ordinary cumulative sentences. Finally, in section 5, this contribution is compared to existing proposals on cumulative readings of *every* and the puzzle of collectivity is discussed.

2 Homogeneity and cumulativity: the implicature approach

This section introduces the notion of *homogeneity* (Löbner, 2000, a.o.) and studies the homogeneity properties of cumulative sentences with quantifiers. The main observation is that the truth-conditions of negative sentences is straightforward to account for if we replace the cumulative stipulation for *open* by an existential stipulation. This stipulation does not derive inferences attested with positive sentences though. However, we'll see that the missed inferences resemble known implicatures, extending Bar-Lev (2018a)'s account of homogeneity. The next section will then formalize these intuitions.

2.1 Homogeneity and the homogeneity properties of cumulative sentences

Homogeneity[8] (Bar-Lev, 2018a,b; Kriz, 2015; Križ, 2016; Križ and Spector, 2021; Löbner, 2000; Magri, 2014; Malamud, 2012; Schwarzschild, 1993) is a well-known property of plural sentences. It refers to the fact that positive plural sentences have quasi-universal truth-conditions while negative sentences have (quasi-)negative universal truth-conditions. For instance, (10a) is roughly equivalent to *every dancer smiled*[9] while (10b) is roughly equivalent to *no dancer smiled*. This leads to truth-value gaps, some scenario, like (10), are neither true of the positive sentence nor of the negative sentence.

(10) **Context:** *Half of the dancers are smiling and the other half is crying*

a. # The ten dancers are smiling.

b. # The ten dancers aren't smiling.

Ordinary cumulative sentences are also homogeneous (Gajewski, 2005; Kriz, 2015). By taking the complement of the cumulative truth-conditions, one may expect (12a) to be true in the circumstances described in (12b). But the observed truth-conditions of (12c) are stronger and

[8] By extension, the name is also used to refer to other truth-value gaps as conditionals (Bassi and Bar-Lev, 2018), embedded questions (Kriz, 2015).

[9] Exceptions are possible ; this is known as *non-maximality* Kriz (2015). We'll discuss non-maximality again in section 2.3.

require that no oysters whatsoever have been opened (Gajewski (2005), ex. 499, Kriz (2015), ex. 17-18).

(11) a. The 10 cooks opened the 15 oysters.

b. **Cumulative truth-conditions:**
Every one of the 10 cooks opened an oyster.
Every one of the 15 oysters was opened by a cook.

(12) a. The 10 cooks didn't open the 15 oysters.

b. **Complement of the cumulative truth-conditions:**
Either not every one of the 10 cooks opened an oyster,
or not every one of the fifteen oysters was opened by a cook.

c. **Attested truth-conditions:**
None of the 10 cooks opened any of the 15 oysters.

Cumulative sentences with *every* also display homogeneity. The negative sentence in (14a) is true in just the case outlined in (14b). By simply taking the complement of the cumulative truth-conditions, one might have expected the truth-conditions to be as in (12b). But these truth-conditions are too weak: the sentence is not simply true when every oyster was opened but some cook didn't contribute to the opening.

(13) a. The 10 cooks opened every oyster.

b. **Truth-conditions:**
Every one of the 10 cooks opened an oyster.
Every oyster was opened by a cook.

(14) a. The ten cooks didn't open every oyster.

b. **Truth-conditions:**
Not every oyster was opened by a cook.

To put it concisely, we may say that (14a) does not deny the exhaustive participation of the cooks, but simply the exhaustive opening of the oysters.

As for *every*, the negation of a cumulative sentence with *most* does not deny the exhaustive participation of the cooks. It simply negates the fact that a majority of oysters were opened.

(15) a. The ten cooks didn't open most oysters.

b. **Truth-conditions:**
The number of oysters opened by a cook is less than half the total number of oysters.

The existence of the homogeneity truth-value gap means that we may not understand semantics of plural sentences by investigating the truth-conditions of positive sentences only. Negative sentences are also important. This insight, as the sequel will show, is important for the problem of cumulative readings. As we'll see, the problem of cumulative readings of *every* simply does not arise in the negative form.

2.2 A simple analysis of the truth-conditions of negative sentences

In section 1, a cumulative stipulation on the verb *open* was made to account for the cumulative reading of ordinary cumulative sentences:

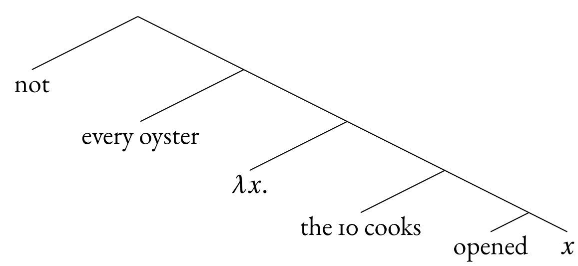
(16) a. The 10 cooks opened the 15 oysters.
b. **Cumulative lexical stipulation:**
⟦opened⟧ $(X)(Y)$ iff
every one of Y opened one of X
and every one of X was opened by one of Y

Because of homogeneity, this assumption is not adequate for negative cumulative sentences, whose truth-conditions are not the complement of the truth-conditions in (16b). To make the observed truth-conditions in (17b) equivalent to the truth-conditions derived by composition in (17c), we would rather need *an existential stipulation* (as in (18)) on the meaning of *open*.

(17) a. The 10 cooks didn't open the 15 oysters.
b. **Observed truth-conditions:**
None of the 10 cooks opened any of the 15 oysters.
c. ⟦(17a)⟧ is true iff $\neg\mathsf{opened}(\iota\mathsf{oysters})(\iota\mathsf{cooks})$

(18) **Existential lexical stipulation:**
⟦opened⟧ $(X)(Y)$ iff
one of Y opened one of X

Setting aside for the moment the question of how the truth-conditions of positive and negative sentences are connected, let us observe that the existential stipulation is not only adequate for the negative counterpart of ordinary cumulative sentences but also for the negative counterpart of cumulative sentences with quantifiers. (19) illustrate this by looking at the case of cumulative readings of *every*: there, the truth-conditions derived simply asserts that not every oysters was opened by a cook, the intuitive reading.

(19) a.


b. The 10 cooks didn't open every oyster.
c. ⟦(19a)⟧ is true
$\Leftrightarrow \neg\forall y \in \mathsf{oyster},\ \mathsf{opened}(y)(\iota\mathsf{cooks})$
(by composition)
$\Leftrightarrow \neg\forall y \in \mathsf{oyster}, \exists x \prec \iota\mathsf{cooks}, \mathsf{opened}(y)(x)$
(by existential lexical stipulation)
= attested truth-conditions

This lexical stipulation also proves adequate for the cumulative readings of *most*. For instance, assuming a simple semantics for *most* as in (20b), we derive the correct truth-conditions in (20c).

(20) a. The 10 cooks didn't open most oysters.

b. $[\![\text{most oysters}]\!](q) = \#\{x \in \mathsf{oyster} \mid \mathsf{q}(x)\} > \frac{1}{2}\#\mathsf{oyster}$

c. $[\![(20a)]\!]$ is true
$\Leftrightarrow \neg\#\{x \in \mathsf{oyster} \middle| \mathsf{open}(x)(\iota\mathsf{cook})\} > \frac{1}{2}\#\mathsf{oyster}$
(by composition)
$\Leftrightarrow \#\{x \in \mathsf{oyster} \middle| \exists y \prec \iota\mathsf{cooks}, \mathsf{open}(x)(y)\} \leq \frac{1}{2}\#\mathsf{oyster}$
(by existential lexical stipulation)
"Less than half the oysters were opened by a cook."
= attested truth-conditions

In summary, we find that, remarkably, the problem of cumulative readings of quantifiers does not arise in the negative form: there, unlike in the positive case, one and the same lexical stipulation can account for both ordinary cumulative sentences and cumulative sentences with quantifiers.

2.3 Missed inferences in positive sentences

Negative cumulative sentences of quantifiers don't raise particular compositional issues. This suggests a strategy to solve the problem raised by their positive counterparts: if we understood how the truth-conditions of positive plural sentences are connected to the truth-conditions of negative plural sentences, we may hope to derive the former from the latter.

Understanding this connection is essentially the problem of homogeneity. Many different proposals for homogeneity exist in the literature (Bar-Lev, 2018b; Kriz, 2015; Križ and Spector, 2021; Malamud, 2012). In what follows, I will argue that Bar-Lev (2018b)'s theory of homogeneity provides exactly what we need: a way of deriving the problematic truth-conditions of positive cumulative sentences with quantifiers from their negative counterparts. What sets Bar-Lev's theory apart from others is that his theory takes the existential meaning embodied in the existential stipulation of section 2.2 as basic and derives the positive meaning as an implicature. As we'll see, the implicature needed in our case is one independently known to exist: a distributive implicature. This is the main reason for picking this account over others[10].

In the sequel, I'll present Bar-Lev (2018b)'s theory superficially, before showing how it may apply to the problem of cumulative sentences.

2.4 Bar-Lev's account of homogeneity and analogy to Free Choice

Bar-Lev (2018b) focuses on non-cumulative examples like (21a) and (21b).

[10] Other accounts of homogeneity could in principle piggy-back on the truth-conditions of negative cumulative sentences with quantifiers to provide an account of their positive counterparts. I leave it to future research to see whether similar accounts may be developed using other theories of homogeneity.

(21) a. The dancers didn't smile.
b. The dancers smiled.

Bar-Lev assumes the verb phrase receives an existential meaning. As in section 2.2, this existential meaning could[11] be the result of an existential lexical stipulation on the verb *smile*, e.g. (22). To highlight that a verb is stipulated to have an existential semantics, I'll write ∃-*smile* (in much the same way that Kratzer (2003) writes $^{*}\mathsf{open}$ to highlight her assumption that lexical denotations are closed under sums).

(22) **Existential lexical stipulation for *smile*:**
$[\![\exists\text{-smile}]\!](X)$ iff
one of Y smiled

With this stipulation, the underlying truth-conditions of the examples in (22) are predicted to be as in (23). This provides adequate results for the negative sentences ; in the positive case, the truth-conditions are lacking the inference in gray, which I'll call a participation inference (i.e. the inference that all dancers smiled).

(23) a. **Truth-conditions of (22a):**
It's not the case that any dancer smiled.
b. **Truth-conditions of (22b):**
Some dancers smiled.
Every dancer smiled.

As is well-known in the literature on homogeneity, the participation inference in gray has a particular status. First, it is not observed in negative sentences. Second, it can be partially cancelled, a phenomenon known as *non-maximality*. For instance, in a circumstance when we want to know the dancers' general mood, the sentence may be true even if there are a few dancers with a neutral expression (Kriz, 2015).

Other accounts of homogeneity offer different ways of thinking about the status of participation inferences[12]. These two properties of participation inferences - polarity sensitivity and (partial or full) cancellability - are also characteristic of implicatures. This motivates Bar-Lev (2018a)'s treatment of participation inferences as implicatures, more precisely as (a form of) Free Choice implicatures.

Indeed, in Free Choice implicatures (illustrated in (24)), a disjunction embedded under an existential modal is interpreted as a wide-scope conjunction.

(24) a. You are allowed to eat apple or cake.
b. **Truth-conditions:**
$\Diamond(\mathsf{cake} \vee \mathsf{apple})$
$\wedge \Diamond\mathsf{cake} \wedge \Diamond\mathsf{apple}$

[11]In his system, the existential meaning is not obtained as a lexical stipulation but as the result of applying an (existential) distributivity operator. In , I argue that not all homogeneity stems from distributivity operators and that at least some stem from the lexical semantics of the verb. For present purposes, the difference appears immaterial.

[12]In Kriz (2015) for instance, the participation inference corresponds to the true worlds. Cancellation occurs when truth-value-less worlds are treated as true. In Križ and Spector (2021), it is obtained as the conjunction of candidate meanings and cancelled when some of these are filtered by the QUD.

Seeing existentials and universals as generalized counterparts of disjunctions and conjunctions, we can see a parallel between Free Choice implicatures and sentences like (25a): in both cases, an existential or disjunctive meaning is converted to a universal or conjunctive meaning[13].

(25) a. The dancers ∃-smiled.
b. $\exists x \prec \iota\mathsf{dancers},\ \mathsf{smiled}(x)$
$\forall x \prec \iota\mathsf{dancers},\ \mathsf{smiled}(x)$

The analogy with Free Choice implicature works as far as simple sentences are concerned. For the case of cumulative sentences, the analogy to Free Choice no longer seems to hold. Consider (26). Following section 2.2, we assume the lexical semantics of *open* to be existential as in (26)[14].

(26) **Existential lexical stipulation:**
$[\![\exists\text{-opened}]\!](X)(Y)$ iff
one of Y opened one of X

With this assumption, the underlying meaning of (27a) will be as in the first line of (27b). Like in the simple case, we're lacking a participation inference that all cooks opened an oyster.

(27) a. The cooks ∃-opened every oyster.
b. **Attested meaning:**
$\forall x \in \mathsf{oyster}, \exists y \prec \iota\mathsf{cooks}, \mathrm{open}(x)(y)$
$\forall y \prec \iota\mathsf{cooks}, \exists y \in [\![\mathrm{oyster}]\!], \mathrm{open}(x)(y)$

Like the participation inferences in the intransitive case, this participation inference is cancellable (i.e. non-maximality is possible) ; certain contexts allow the sentence to be true even when this inference does not hold. By contrast, there is no situation where the inferences in black are not met and the sentence is still assertable. This is illustrated by (28): a continuation denying the predicted inference (e.g. (28a)) is odd but a continuation denying the missed inference is acceptable.

(28) The cooks on staff that night opened every oyster in this bag ...
a. # Of course, following a widespread superstition, they didn't open the last three oysters.
b. Of course, the more experienced cooks, as usual, found excuses not to do anything so it was only the rookies.

[13] This analogy is somewhat loose for Free Choice, since the typical free choice inference only occurs in the scope of a possibility modal (cf (24)), and the universal inference we observed can occur in the absence of a modal. Still, this configuration has been argued to *not* be critical for Free Choice-like implicatures: the strengthening of unembedded disjunctions to conjunctions has proven useful to account for properties of Warlpiri connectives *manu* (Bowler, 2014) and children's conjunctive interpretation of *or* (Singh et al., 2016).

[14] More generally, I speculate that verbs that have this underlying existential meaning like *open* are the ones that partake in cumulative readings. Some verbs like *outweigh* don't seem to yield cumulative readings. We may assume these verbs are not subject to the existential denotation. This predicts, correctly, that they do not have homogeneneity. This prediction is explored in more depth in Chatain (2021b)

Yet, this participation inference is not a Free Choice inference. The assumed underlying meaning asserts that all oysters were opened by a cook and the participation inference to be derived asserts that all cooks opened an oyster. Like Free Choice inferences, an existential in the underlying meaning (*an oyster*) is indeed converted to a universal, but in addition, the universal meaning of *every* in the underlying meaning is "*weakened*" to an existential.

However, another type of implicatures gives an adequate parallel: distributive implicatures. Distributive implicatures occur when a disjunction is embedded under some quantifiers such as *every* in (29). In this case, an implicature arise that every disjunct is true of at least one individual in the domain of the quantifier.

(29) Every ambassador speaks Arabic, English or Mandarin.
$\rightsquigarrow$ at least one ambassador speaks Arabic.
$\rightsquigarrow$ at least one ambassador speaks English.
$\rightsquigarrow$ at least one ambassador speaks Mandarin.

This inference has a similar shape to the participation inference of cumulative sentences. To see this, consider a scenario in which there are only three cooks: Joana, Marius and Becky. In this case, the underlying meaning of (27a) simplifies to (30a) and the participation inference to be derived can be written as in (30b). The inferences are isomorphic to the inferences in (29).

(30) a. $\forall x \in \mathsf{oyster}, \mathsf{open}(x)(\mathsf{Joana}) \lor \mathsf{open}(x)(\mathsf{Marius}) \lor \mathsf{open}(x)(\mathsf{Becky})$
$\rightsquigarrow$ *every oyster was opened by Joana, Marius or Becky*
b. $\exists x \in \mathsf{oyster}, \mathsf{open}(x)(\mathsf{Joana})$
$\exists x \in \mathsf{oyster}, \mathsf{open}(x)(\mathsf{Marius})$
$\exists x \in \mathsf{oyster}, \mathsf{open}(x)(\mathsf{Becky})$

The parallel does not stop at cumulative readings of *every*. Other quantifiers as well give rise to distributive implicatures. For instance, non-partitive *most*[15]:

(31) Most ambassadors speak Arabic, English or Mandarin.
$\rightsquigarrow$ at least one ambassador speaks Arabic.
$\rightsquigarrow$ at least one ambassador speaks English.
$\rightsquigarrow$ at least one ambassador speaks Mandarin.

Correspondingly, these inferences match the participation inferences of cumulative sentences with *most*:

(32) a. The cooks ∃-opened most oysters.
b. $\mathsf{Most}\ x \in \mathsf{oyster}, \mathsf{open}(x)(\mathsf{Joana}) \lor \mathsf{open}(x)(\mathsf{Marius}) \lor \mathsf{open}(x)(\mathsf{Becky})$
$\rightsquigarrow$ *most oysters were opened by Joana, Marius or Becky*
c. $\exists x \in \mathsf{oyster}, \mathrm{open}(x)(\mathsf{Joana})$
$\exists x \in \mathsf{oyster}, \mathrm{open}(x)(\mathsf{Marius})$
$\exists x \in \mathsf{oyster}, \mathrm{open}(x)(\mathsf{Becky})$

[15] Credits for this observation goes to F. Hisao Kobayashi (p.c.).

2.5 Summary

The problem of cumulative readings of quantifiers remarkably does not arise in negative sentences: there, assuming only a weak lexical meaning and standard composition derives the truth-conditions of both ordinary cumulative sentences and cumulative readings of *every*.

This existential stipulation however isn't sufficient for positive sentences. It will systematically miss the participation inferences. Following Bar-Lev (2018b), we take participation inferences to be implicatures. Like them, they are polarity-sensitive and cancellable. These implicatures parallel ones independently attested, namely Free Choice implicatures for non-cumulative sentences and distributive implicatures for cumulative sentences. This formal parallel is summarized in the chart below. In the next section, a theory that captures these implicatures is provided and then applied to the cumulative case.

Sentence	You can have ice-cream or cake.	The dancers smiled.
Underlying	$\Diamond a \vee \Diamond b$	$\exists x \prec \iota\mathsf{dancers}, \mathsf{smiled}(x)$
Implicature	$\Diamond a \wedge \Diamond b$	$\forall x \prec \iota\mathsf{dancers}, \mathsf{smiled}(x)$
Sentence	Everyone speaks Mandarin or Tagalog.	The cooks opened every oyster.
Underlying	$\forall x, m(x) \vee t(x)$	$\forall y \in \mathsf{oyster}, \exists x \prec \iota\mathsf{cooks}, \mathsf{opened}(x)(y)$
Implicature	$\exists x, m(x) \wedge \exists x, t(x)$	$\forall x \prec \iota\mathsf{cooks}, \exists y \in \mathsf{oyster}, \mathsf{opened}(x)(y)$

In a nutshell, the intuition behind this account is that there is a connection between homogeneity and cumulativity. It assumes that both are made possible by weak existential meanings, which are subsequently strengthened. Note that I do not claim that cumulative readings always go with homogeneous truth-conditions, as observed on the surface. Section 4.4 will discuss cases of homogeneity cancellation, where some elements intervene to mask the underlying existential meaning of the verb. Yet, such elements do give rise to cumulativity.

3 Accounting for participation inferences: recursive exhaustification

This section presents the implicature account of participation inferences in non-cumulative and cumulative sentences. We start from the unified account of Free Choice/distributive implicature found in Bar-Lev and Fox (2017); Fox (2007) and extend it *mutatis mutandis* to the plural sentences.

Following the grammatical tradition (Chierchia, 2013; Chierchia et al., 2012; Fox, 2007), this account assumes that the scalar implicatures of interest are derived in the grammar by application of a covert operator Exh. This operator strengthens the meaning of a prejacent by comparing it to a set of alternatives. I specifically assume the *innocent exclusion exhaustification* of Fox (2007), defined below in (33). To put it concisely, this operator negates as many alternatives as possible while (i) not creating logical contradictions and (ii) treating all alternatives symmetrically.

(33) a. **Maximal sets:**

$$\mathrm{Max}(p)(\mathsf{alts}) := \left\{ S \subseteq \mathsf{alts} \;\middle|\; \begin{array}{l} p \wedge \bigwedge_{\mathsf{alt} \in S} \neg\mathsf{alt} \text{ is not contradictory} \\ \neg\exists S' \supseteq S, p \wedge \bigwedge_{\mathsf{alt} \in S'} \neg\mathsf{alt} \text{ is not contradictory} \end{array} \right\}$$

b. **Innocently excludable alternatives:**

$IE(p)(\mathsf{alts}) := \bigcap \mathrm{Max}(p)(\mathsf{alts})$

c. $[\![\mathrm{Exh}_{\ \mathrm{alts}}]\!]\,(p) = p \wedge \bigwedge_{\mathsf{alt} \in \mathsf{IE}(p)(\mathsf{alts})} \neg \mathsf{alt}$

(34) illustrates how EXH can derive the *not-and* implicature of simple disjunctions.

(34) a. EXH [Joana or Marius came]

b. **Alternatives:**
   - $\mathsf{came(Joana)}$
   - $\mathsf{came(Marius)}$
   - $\mathsf{came(Joana)} \wedge \mathsf{came(Marius)}$

c. **Maximal sets:**
   - $\{\mathsf{came(Joana)}, \mathsf{came(Joana)} \wedge \mathsf{came(Marius)}\}$
   - $\{\mathsf{came(Marius)}, \mathsf{came(Joana)} \wedge \mathsf{came(Marius)}\}$

d. **Innocently excludable alternatives:**
$\{\mathsf{came(Joana)} \wedge \mathsf{came(Marius)}\}$

In this section, I show, following previous literature, how applying this operator recursively can derive Free Choice and distributive implicatures. I then proceed to adapt the account to sentences with plural arguments.

3.1 Inclusivity and Free Choice

The Free Choice implicature, repeated in (35), cannot be derived by applying one EXH operator as we did above for ordinary disjunctions. The Free Choice inferences of (35b) in gray could only be obtained as the negation of "*You're not allowed to eat apples*" or the negation of "*You're not allowed to eat cake*". But these sentences are not alternatives to the sentence.

(35) a. You are allowed to eat apples or cake.

b. **Paraphrase:**
You're allowed to eat apples or cake
You're allowed to eat apples
You're allowed to eat cake

This informal reasoning suggests that something extra is needed to derive Free Choice implicatures. Fox (2007) proposes to derive these implicatures by applying two EXH operators (i.e. recursive exhaustification). Specifically, he assumes the sentence (35) has the structure in (36). I will use [a] and [b] as labels to differentiate the two operators.

(36) $\mathrm{EXH}_{[b]}$ $\mathrm{EXH}_{[a]}$ you are allowed to eat apple or cake.

To guide intuitions about what this is meant to accomplish, we can reason along the following Gricean lines (cf. Kratzer and Shimoyama (2017)): the speaker chose to assert "apples or cake" and not either of the disjuncts. This means she was probably not in a position to assert one of the disjuncts. One reason might be that by uttering e.g. (37a), she may have conveyed that "apple" was the only option (by way of implicature), likewise for (37b). From this, we conclude that the speaker believes neither "apple" nor "cake" is the only option, i.e. both are in fact allowed.

(37) a. You are allowed to eat apples.
$\rightsquigarrow$ *you're allowed to eat apples and nothing else*

b. You are allowed to eat cake.
$\rightsquigarrow$ *you're allowed to eat cake and nothing else*

This reasoning relied on negating alternatives (e.g. (37a) and (37b)) *along with their implicatures*. In a grammatical tradition, this means negating alternatives which themselves contain an Exh operator, something that can be achieved with the recursive structure in (36).

To compute the truth-conditions that such a structure yields, one needs to compute the result of applying $\textsc{Exh}_{[b]}$ to a structure like "*$\textsc{Exh}_{[a]}$ you are allowed to eat apples or cake*". This means comparing the sentence in (38a) to alternatives of the form (38b).

(38) a. **Prejacent:**
$\textsc{Exh}_{[a]}\left(\Diamond(\mathsf{cake} \vee \mathsf{apples}), \mathsf{alts}_1\right)$
where $\mathsf{alts}_1 = \{\Diamond\mathsf{cake}, \Diamond\mathsf{apples}\}$

b. **Alternatives for $\textsc{Exh}_{[b]}$:**
$\mathsf{alts}_2 = \{\textsc{Exh}_{[a]}(\Diamond\mathsf{cake}, \mathsf{alts}_1), \textsc{Exh}_{[a]}(\Diamond\mathsf{apples}, \mathsf{alts}_1)\}$

These alternatives in (39b) are themselves exhaustive statements. By assumption, Fox (2007) assume that these alternatives are all exhaustified with respect to the same set of alternatives, the alternatives to "*You are allowed to eat apples or cake*" (called alt_1 in (38)). The exhaustified alternatives have the meaning given in (39), which both assert that some option (*apples* or *cake*) is the only allowed option.

(39) **Alternatives for $\textsc{Exh}_{[a]}$:**
$\mathsf{alts}_1 = \{\Diamond\mathsf{cake}, \Diamond\mathsf{apples})\}$

a. $\textsc{Exh}_{[a]}(\Diamond\mathsf{cake}, \mathsf{alts}_1) = \Diamond\mathsf{cake} \wedge \neg\Diamond\mathsf{apples}$
$\rightsquigarrow$ *you're allowed to eat cake and not apples*

b. $\textsc{Exh}_{[a]}(\Diamond\mathsf{apples}, \mathsf{alts}_1) = \Diamond\mathsf{apples} \wedge \neg\Diamond\mathsf{cake}$
$\rightsquigarrow$*you're allowed to eat apples and not cake*

Negating these alternatives yields the attested FC inference that both options are in fact allowed:

(40)
$$\begin{aligned}
&\textsc{Exh}_{[b]}(\textsc{Exh}_{[a]}(\Diamond(\mathsf{cake} \vee \mathsf{apples}, \mathsf{alts}_1)))\\
&= \textsc{Exh}_{[a]}(\Diamond(\mathsf{cake}\vee\mathsf{apples}), \mathsf{alts}_1) \wedge \neg\textsc{Exh}_{[a]}(\Diamond\mathsf{cake}, \mathsf{alts}_1) \wedge \neg\textsc{Exh}_{[a]}(\Diamond\mathsf{apples}, \mathsf{alts}_1)\\
&= \Diamond\left(\mathsf{cake} \vee \mathsf{apples}\right) \wedge \neg\left(\Diamond\mathsf{cake} \wedge \neg\Diamond\mathsf{apples}\right) \wedge \neg\left(\Diamond\mathsf{apples} \wedge \neg\Diamond\mathsf{cake}\right)\\
&= \Diamond\mathsf{cake} \wedge \Diamond\mathsf{apples}
\end{aligned}$$

(41) $\textsc{Exh}_{[b]}$ $\textsc{Exh}_{[a]}$ You ate apple or cake.

a. You ate apple.
$\rightsquigarrow$ *You only ate apple.*

b. You ate cake.
$\rightsquigarrow$ *You only ate cake.*

This reasoning offers a mechanism to turn disjunctive meanings into conjunctive meanings. It may seem that applying the same reasoning to the simple unembedded disjunction would derive that you ate both apple and cake, which would be incorrect. In unembedded cases, it is assumed that the exclusive inference (*not and*) in $\textsc{Exh}_{[a]}$ blocks the Free Choice inference. This safeguard does not work when a modal intervenes, as in the case given, or when there is no *and* implicature, as in the plural case we now turn to. The sequel shows how one can adapt Bar-Lev (2018b)'s recipe to the plural case and derive an account of participation inferences in positive sentences.

Applying Free Choice Reasoning to plural sentences Recall the non-cumulative sentence in (42a). As seen in section 2, Bar-Lev (2018b) assumes that the truth-conditions of the whole sentence, prior to exhaustification by Exh, are existential, as in (42a).

(42) a. The dancers smiled.
b. **Unstrengthened meaning:**
$\exists x \prec \iota\text{dancers},\ \text{smiled}(x)$
c. **Attested meaning:**
$\exists x \prec \iota\text{dancers},\ \text{smiled}(x)$
$\wedge \forall x \prec \iota\text{dancers},\ \text{smiled}(x)$

To make this case completely parallel to the case of Free Choice and deliver the attested universal truth-conditions, Bar-Lev (2018b) needs a counterpart to the "*individual disjunct*"[16] alternatives of disjunction, seen in (41a) and (41b). Thinking of an existential as a grand disjunction, as in (43), these alternatives find a parallel in the *sub-domain alternatives* of the existential: alternatives where the existential is constrained to range over a smaller set of entities. In our specific case, where the domain of the existential are the atomic parts of *the dancers*, the sub-domain alternatives are simply sentences where *the dancers* is replaced by a plurality of smaller size X[17], as in (43). Crucially, *smile* is assumed to not have a universal alternative, a putative verb *smyle* with universal truth-conditions. As seen in the previous section, having no conjunctive or universal counterpart is what allows the Free Choice mechanism to proceed[18].

[16] Bar-Lev (2018b) uses Innocent Inclusion, a new mechanism of exhaustification, to derive Free Choice, not recursive exhaustification. However, Innocent Inclusion derives too strong a distributive implicature, so it can't be used here, hence my choice of recursive exhaustification. As a reviewer notes, this might create a dilemma: there are arguments that the Universal Free Choice inference (from *every child is allowed ice-cream or cake* to *every child is allowed cake*) ought to be derived globally (Bar-Lev and Fox, 2017; Chemla, 2009). If the modal Free Choice is an indication of what an embedded disjunction without conjunctive alternative, then cumulative sentences, an instance of this, ought to receive Universal Free Choice-like inferences, not distributive inferences. So either we reject the idea that Universal Free Choice requires a global derivation or we must find another way to the strengthening necessary for cumulativity. I leave a solution of this dilemma to future research.

[17] Similar alternatives are needed in the literature on exceptives (Crnič, 2018; Hirsch, 2016).

[18] *smyle* may not exist but a reviewer notes a problem: there may be universal alternatives independently. For instance, in more complex sentences like *the dancers had a pie together* (modified from reviewer's original example), *the dancers had a pie each* might be an alternative. It would be obtained by substituting *together* with *each*. If it is, then the participation inferences may not be generated at the matrix level. However, they may still be generated in embedded positions if the structure is (43a) or (43b), as we'll discuss in section 4.

(44) a. $\exists x \prec \iota\text{dancers}, \text{smiled}(x)$
$\leftrightsquigarrow \text{smiled(dancer 1)} \vee \text{smiled(dancer 2)} \vee \text{smiled(dancer 3)} \vee \ldots$

b. $\exists x \prec X, \text{smiled}(x)$
where $X \prec \iota\text{dancers}$

> **Assumptions about alternatives (to be modified)**
>
> 1. ⟦the NP⟧ has as alternatives all pluralities X, such that $X \prec$ ⟦the NP⟧

Everything is in place to derive the universal inference of the non-cumulative sentence in (42). The recursive exhaustification structure is given in (45).

(45) $\text{Exh}_{[b]}$ $\text{Exh}_{[a]}$ [The dancers ∃-smiled.]$_\alpha$
Alts. to α:
$\{⟦\exists\text{-smiled}⟧(X) \mid X \prec ⟦\text{the dancers}⟧\}$

By recursive exhaustification, (45) will be strengthened to a universal meaning, just as in the Free Choice case (details in (46)). Informally, the alternatives to the constituent that $\text{Exh}_{[b]}$ heads can be paraphrased as *among the dancers, only X smiled*, where X is strict sub-plurality of *the dancers*. They can all be negated consistently by $\text{Exh}_{[a]}$; negating these alternatives is equivalent to asserting that either all the dancers smiled or none of them did. Together with the prejacent, this entails that all the dancers smiled, the desired result. (Reminder that I use ∃-smile in the the LF representations below to emphasize the existential stipulation associated with the verb.)

(46) a. **Alternatives to** $\text{Exh}_{[a]}$ [The dancers ∃-smiled.]$_\alpha$:
- Only Marie-Lou ∃-smiled.
- Only the dancers who are not Marie-Lou ∃-smiled.
- Only Marius ∃-smiled.
- Only the dancers who are not Marius ∃-smiled.
- …

b. **Implicatures generated by** $\textbf{Exh}_{[b]}$**:**
1. Not only Marie-Lou ∃-smiled.
$\rightsquigarrow$ either Marie-Lou didn't smile or someone who wasn't Marie-Lou smiled.
2. Not only the dancers who aren't Marie-Lou ∃-smiled.
$\rightsquigarrow$ either Marie-Lou smiled or no one who wasn't Marie-Lou smiled.
$\rightsquigarrow$ either (Marie-Lou smiled and someone other than her did as well) or no one smiled*(together with 1)*
3. …

This result is only approximate. As alluded to earlier, the participation inferences are not always strongly universal: if context allows for it, the sentence may be uttered even if some

(43) a. [[the dancers had a pie] together]
b. [the dancers [t had a pie] together]

dancers are not smiling. This is the phenomenon of *non-maximality* (Brisson, 1997; Kriz, 2015; Malamud, 2012).

This paper does not deal with non-maximality ; however, it is useful to give a glance on how such effects can be accommodated within the implicature theory of Bar-Lev (2018a). Following this work, it is assumed that non-maximal readings arise when some alternatives are pruned, i.e. removed from the set of alternatives. This is the same mechanism that underlies regular implicature cancellation in a grammatical theory (Fox, 2007).

Suppose, for instance, that alternatives where the subject denotes a singularity are excluded from the set of alternatives. Then, the alternatives compared by $\textsc{Exh}_{[a]}$ are of the form in (47a), generating the implicature in (47b). Combining the implicatures together with the meaning of the prejacent yields the reading that either every dancer but one smiled or no one smiled. This is one of many non-maximal readings.

(47) a. **Alternatives to** $\textsc{Exh}_{[b]}$ [The dancers ∃-smiled.]$_\alpha$:
- Only Marie-Lou and Marius ∃-smiled.
- Only the dancers who are neither Marie-Lou nor Marius ∃-smiled.
- Only Marius and Bart ∃-smiled.
- Only the dancers who are neither Marius nor Bart ∃-smiled.
- …

b. **Implicatures generated by $\textsc{Exh}_{[b]}$:**
1. Not only Marie-Lou and Marius ∃-smiled.
   ⇝ either neither Marie-Lou nor Marius smiled or someone else smiled.
2. Not only the dancers who aren't Marie-Lou or Marius ∃-smiled.
   ⇝ either Marie-Lou or Marius smiled or no one else smiled.
   ⇝ either (Marie-Lou or Marius smiled and someone else did as well) or no one smiled *(together with 1)*
3. …

This is only a cursory outlook on non-maximality. The interested reader can refer to Bar-Lev (2018a). In the sequel, I will focus on maximal readings ; an account in terms of pruning can explain the non-maximal readings.

3.2 Participation inferences in cumulative readings of *every* and distributive implicatures

The same recursive mechanism can account for distributive implicatures and participation inferences in cumulative sentences. As we saw, the participation inferences of cumulative sentences we are trying to account for, repeated in (48a), are formally similar to distributive implicatures, (48b).

(48) a. The cooks ∃-opened every oyster.
⇝ Susan opened at least one oyster.
⇝ Adrian opened at least one oyster.
⇝ Walter opened at least one oyster.

b. Every ambassador speaks Arabic, English or Mandarin.
⇝ at least one ambassador speaks Arabic.

⇝ at least one ambassador speaks English.
⇝ at least one ambassador speaks Mandarin.

As with Free Choice, let me start by motivating the use of recursive exhaustification in an account of distributive implicatures. Traditionally[19], distributive implicatures are obtained by negating alternatives where the disjunction "*Arabic, English or Mandarin*" is replaced by a smaller one, as in (49b). The inferences generated by negating such alternatives, together with the prejacent, correctly entail that at least one ambassador speaks Arabic, at least one English, etc.

(49) a. EXH Every ambassador speaks Arabic, English or Mandarin.

b. **Negated alternatives:**

- not every ambassador speaks Mandarin or English [α]
  ⇝ *some ambassador doesn't speak Mandarin or English*
- not every ambassador speaks Arabic or English
  ⇝ *some ambassador doesn't speak Arabic or English*
- not every ambassador speaks Arabic or Mandarin
  ⇝ *some ambassador doesn't speak Arabic or Mandarin*
- not every ambassador speaks Mandarin
  ⇝ *some ambassador doesn't speak Mandarin*
- …

As Crnič et al. (2015) notes on similar examples, it also predicts, incorrectly, that one ambassador *only* speaks Arabic: this is obtained by combining the inference [α] that one ambassador speaks neither Mandarin or English with the inference that every ambassador speaks one of the three languages. Yet, they observe, the sentence can be uttered even when all ambassadors are bilingual in two of the languages, so long as all languages are spoken by at least one ambassador[20]. In addition, Crnič et al. (2015) provide experimental evidence against the existence of strong implicatures such as (49b). Rather, the distributive implicatures derived by speakers appear to be neither stronger nor weaker than what is given in (50).

(50) Every ambassador speaks Arabic, English or Mandarin.
⇝ *some ambassador speaks Arabic*
⇝ *some ambassador speaks English*
⇝ *some ambassador speaks Mandarin*

Crnič et al. (2015)'s problem shows that the traditional account, which derives distributive implicatures from simply negating plain alternatives, is inadequate. The reasons for this failure is that the distributive implicatures, like the Free Choice implicatures, are positive (cf (51)). It is not obvious how to derive them as the negation of an alternative to the sentence, or even a conjunction of such alternatives.

(51) Every ambassador speaks Arabic, English or Mandarin.
⇝ at least one ambassador speaks Arabic.

[19] In either the Gricean tradition (Sauerland, 2004) or the grammatical tradition.

[20] And not every ambassador speaks all three languages. The latter inference comes from competition with *and*, which I don't show here.

$\rightsquigarrow$ at least one ambassador speaks English.
$\rightsquigarrow$ at least one ambassador speaks Mandarin.

Just like Free Choice then, these implicatures seem to require something beyond simple exhaustification. As Bar-Lev and Fox (2016) show, recursive exhaustification can be applied to this case as well. Specifically, they propose the structure below:

(52) $\textsc{Exh}_{[b]}$ $\textsc{Exh}_{[a]}$ every ambassador speaks Arabic, English or Mandarin.

As seen with Free Choice, the idea behind the use of recursive exhaustification is to have a sentence negate not the alternatives to the prejacent, but the alternatives *along with their implicatures*. To see how this helps, consider how hearers may interpret (53), one of the alternatives to the prejacent in (52), when it is relevant which of three languages are spoken among the ambassadors.

(53) *Of these three languages, which are spoken by the ambassadors?*
Every ambassador speaks Arabic or Mandarin.
$\rightsquigarrow$ no ambassador speaks English.

By uttering (54), the speaker seems to convey that the other language, English, is not spoken at all. This is an implicature of the alternative. Recursive exhaustification asserts that the conjunction of (53) with the implicature just described is false ; that is to say that either not every ambassador speaks Arabic or Mandarin or some ambassador speaks English. Either case entails that some ambassador speaks English, which is one of the desired distributive implicatures.

Note that the implicature of the alternative in (53) is derived by negating "*some ambassador speaks English*". It must be assumed that "*some ambassador speaks English*" is an alternative to (53) and *ipso facto* of the original sentence (recall from 3.1 that in recursive exhaustification, the prejacent's alternatives are used as the alternatives' alternatives). This alternative can be obtained by replacing *every* with *some* and simplifying the disjunction "*Arabic, English or Mandarin*" to just the second disjunct "*English*". In particular, the *some/every* scale is a necessary ingredient of this computation. Though standard, I will list this assumption because it will be generalized to other quantifiers, in section 4.1.

Assumptions about alternatives (to be modified)

1. $[\![\text{the NP}]\!]$ has as alternatives all pluralities X, such that $X \prec [\![\text{the NP}]\!]$
2. *every* has *some* as an alternative

Application of cumulative reading of *every*. Let us see how the intuitions developed above are rendered formally. Let us assume the cooks consist of Susan and Adrian. We will consider (54a), the distributive implicature case, and (54b), the cumulative case, in parallel. Indeed, by our assumptions, in both cases, the prejacent asserts that every oyster was opened by either Susan or Adrian. The alternatives for $\textsc{Exh}_{[b]}$ are listed in (55) for both the distributive implicature case and the cumulative case. I separate them between *existential alternatives*, where *every* is replaced by *some* and *universal alternatives*, where *every* is left as is. The meaning of the alternative for $\text{Exh}_{[b]}$, which we derive below, is given in prose.

(54) a. $\text{Exh}_{[b]}$ $\text{Exh}_{[a]}$ the cooks ∃-opened every oyster
b. $\text{Exh}_{[b]}$ $\text{Exh}_{[a]}$ every oyster was opened by Susan or Adrian
c. **Attested implicatures:**
⇝ Susan opened at least one oyster.
⇝ Adrian opened at least one oyster.

(55) a. **Universal alternatives:**

cumulative sentence (54a)	***every*-over-*or*** (54b)
$\text{Exh}_{[a]}$(S ∃-opened every oyster.)	$\text{Exh}_{[a]}$(every oyster was opened by S)
= *every oyster was opened by S and no oyster was opened by A*	
$\text{Exh}_{[a]}$(A ∃-opened every oyster.)	$\text{Exh}_{[a]}$(every oyster was opened by A)
= *every oyster was opened by A and no oyster was opened by S*	

b. **Existential alternatives:**

cumulative sentence (54a)	***every*-over-*or*** (54b)
$\text{Exh}_{[a]}$(S and A ∃-opened some oyster.)	$\text{Exh}_{[a]}$(some oyster was opened by S or A)
= *some but not all oysters were opened by S or A*	
$\text{Exh}_{[a]}$(S ∃-opened some oyster.)	$\text{Exh}_{[a]}$(some oyster was opened by S)
= *some but not all oysters were opened by S and no oyster was opened by A*	
$\text{Exh}_{[a]}$(A ∃-opened some oyster.)	$\text{Exh}_{[a]}$(some oyster was opened by A)
= *some but not all oysters were opened by A and no oyster was opened by S*	

The universal alternatives in (55a) entail that the individual not mentioned by the alternative did not open any oyster. The derivation of this meaning is illustrated in (56) for the universal alternative $\text{Exh}_{[a]}$(S ∃-opened every oyster.) and its counterpart $\text{Exh}_{[a]}$(every oyster was opened by S). This meaning requires looking at the alternatives for $\text{Exh}_{[a]}$; they are listed in (56b). All the alternatives for $\text{Exh}_{[a]}$ marked ¬ can consistently be negated.

(56) $\text{Exh}_{[a]}$(S ∃-opened every oyster.)
$\text{Exh}_{[a]}$(every oyster was opened by Sue.)

a. **Alternatives:**

S and A ∃-opened every oyster	every oyster was opened by S or A	(¬)
A ∃-opened every oyster	every oyster was opened by A	(¬)
S and A ∃-opened some oyster	some oyster was opened by S or A	
S ∃-opened some oyster	some oyster was opened by S	
A ∃-opened some oyster	some oyster was opened by A	(¬)

b. **Resulting meaning:**
Susan opened every oyster and Adrian didn't open any oyster

This derivation captures the intuition already seen in (53) that in a context where it is relevant what was opened by whom, not mentioning *Adrian* will implicate that Adrian didn't open any oyster[21].

(57) Every oyster was opened by Sue.
$\rightsquigarrow$ *no oyster were opened by Adrian*

The existential alternatives in (55b) assert that some but not all oysters were opened by an individual and any individual not mentioned in the alternative didn't open an oyster. This is shown in (58) for $\textsc{Exh}_{[a]}$ (S ∃-opened some oyster) and its counterpart $\textsc{Exh}_{[a]}$(some oyster was opened by S). All the alternatives for $\textsc{Exh}_{[a]}$ marked ¬ can consistently be negated.

(58) $\textsc{Exh}_{[a]}$(S ∃-opened some oyster.)
$\textsc{Exh}_{[a]}$(some oyster was opened by S)

a. **Alternatives:**

S and A ∃-opened every oyster	every oyster was opened by S or A	(¬)
A ∃-opened every oyster	every oyster was opened by A	(¬)
S ∃-opened every oyster	every oyster was opened by S	(¬)
S and A ∃-opened some oyster	some oyster was opened by S or A	
A ∃-opened some oyster	some oyster was opened by A	(¬)

b. **Resulting meaning:**
Susan opened some but not all oysters and Adrian didn't open any oyster

Having ascertained their meaning, we can now see that both the existential and the universal alternatives can be negated by $\textsc{Exh}_{[b]}$. The existential alternatives in (55b) contradict the prejacent, since they implicate that not all oysters were opened. As such, they can always be negated but negating them does not generate any implicature.

The universal alternatives in (55a) can also be negated. Negating them generates the participation inferences. As seen above, the universal alternatives each convey that one of Susan and Adrian opened every oyster while the other didn't open any. Thus, if either cook didn't open any oyster, one of the two universal alternatives would be true. By negating them, we generate the desired inference that both must have opened oysters.

(59) a. $\textsc{Exh}_{[b]}$ $\textsc{Exh}_{[a]}$ the cooks ∃-opened every oyster
$\textsc{Exh}_{[b]}$ $\textsc{Exh}_{[a]}$ every oyster was opened by Susan or Adrian

b. **Predicted implicatures:**
$\rightsquigarrow$ Susan opened at least one oyster.
$\rightsquigarrow$ Adrian opened at least one oyster.

4 Extending to asymmetries, ordinary cumulative sentences and other quantifiers

The last section presented an account of participation inferences in simple non-cumulative sentences and cumulative readings of *every*, modeled after a similar account of Free Choice

[21] This is already entailed by world knowledge that oysters can't be opened twice. (53) remains the better example, since languages can be spoken by more than one individual.

and distributive implicatures respectively. We assumed recursive exhaustification at the root of the tree and that alternatives were constructed following the principles below.

Assumptions about alternatives

1. $[\![\text{the NP}]\!]$ has as alternatives all pluralities X, such that $X \prec [\![\text{the NP}]\!]$
2. "*every*" has *some* as an alternative

However, only a limited portion of our dataset has been covered by the analysis. In this section, we derive participation inferences for other quantifiers than *every*, for ordinary cumulative sentences, bare numeral sentences and provide an account of subject/object asymmetries. Doing so will give the opportunity to flesh out some more assumptions about alternatives and the position of exhaustification.

4.1 What about other quantifiers?

Upward-entailingness entails participation. The existential denotation of *open* also predicted too weak a meaning for other quantifiers than *every*:

(60) a. The 10 cooks opened most oysters.
b. **Truth-conditions:**
Most oysters was opened by a cook.
Every one of the 10 cooks opened an oyster

(61) a. The 10 cooks opened many oysters.
b. **Truth-conditions:**
Many oysters was opened by a cook.
Every one of the 10 cooks opened an oyster

These participation inferences mirror the distributive implicatures of the corresponding sentences:

(62) a. Most oysters were opened by Susan, Adrian or Walter.
$\rightsquigarrow$ *Susan opened an oyster*
$\rightsquigarrow$ *Adrian opened an oyster*
$\rightsquigarrow$ *Walter opened an oyster*
b. Many oysters were opened by Susan, Adrian or Walter.
$\rightsquigarrow$ *Susan opened an oyster*
$\rightsquigarrow$ *Adrian opened an oyster*
$\rightsquigarrow$ *Walter opened an oyster*

Recursive exhaustification does derive the missing inference in all three cases above. Let me illustrate on the case of *most*. Just as with *every*, it must be assumed that *most* has *some* as an alternative[22] and that the cumulative sentence is parsed with two Exh, as in (63a). Some of the critical alternatives to $\text{Exh}_{[b]}$ are given in (63b).

[22] Such an alternative is needed to account for indirect implicatures: *Joana didn't open most oysters* $\rightsquigarrow$ *Joana opened some oysters.*

(63) a. $\text{Exh}_{[b]}$ $\text{Exh}_{[a]}$ [the 10 cooks] ∃-opened most oysters

b. **Alternatives:**

- $\text{Exh}_{[a]}$ (Susan and Walter ∃-opened most oysters)
  ≈ *most oysters were opened by Susan or Walter and none by Adrian*
- $\text{Exh}_{[a]}$ (Adrian and Walter ∃-opened most oysters)
- $\text{Exh}_{[a]}$ (Susan and Adrian ∃-opened most oysters)
- $\text{Exh}_{[a]}$ (Susan and Walter ∃-opened some oysters)
  ≈ *some but not most oysters were opened by Susan or Walter and none by Adrian*
- $\text{Exh}_{[a]}$ (Adrian and Walter ∃-opened some oysters)
- $\text{Exh}_{[a]}$ (Susan and Adrian ∃-opened some oysters)
- $\text{Exh}_{[a]}$ (Susan, Adrian and Adrian ∃-opened some oysters)
  ≈ *some but not most oysters were opened by Susan or Walter or Adrian*

All alternatives to the sentence can be negated. Indeed, these alternatives are all false in a world where all three cooks opened an oyster, as can be seen from the paraphrases provided. Reciprocally, in any world where all these alternatives are false and the prejacent is true, all cooks opened an oyster. To see this, consider what would happen if Adrian, one of the cooks, didn't open any oyster. Then it would be true that most oysters were opened by Susan or Walter, since the prejacent assert that most oysters were opened by one of the cooks and we know Adrian didn't contribute to the collective effort. That would make the first alternative of (64b) true. Because this alternative is innocently excludable, we know that it can't possibly be true. By *reductio ad absurdum*, we show that Adrian must have opened an oyster. By symmetry, all cooks must have opened an oyster.

More generally, one can prove that recursive exhaustification will derive participation inferences for upward-entailing quantifiers, provided they have *some* as an alternative:

The "*UE entails participation*" guarantee.

Let $\mathcal{Q}$ be a non-trivial quantifier, $\exists_C$ an existential quantifier with sub-domain alternatives.
If the following conditions hold:
- $\mathcal{Q}$ is upward-entailing,
- $\mathcal{Q}$ has *some* as its *only* alternative

then $\text{Exh}_{[b]}$ $\text{Exh}_{[a]}(\mathcal{Q}x, \exists_C y, R(x,y))$ will be equivalent to the conjunction of the prejacent and the inference that $\forall y, \exists x, R(x,y)$

The formal proof of the result is complex and left out for space reasons ; the interested reader can find it in Chatain (2021b). Provided we ignore other alternatives these quantifiers might have, this result guarantees that all of the sentences in (64) will yield the participation inferences:

(64) a. The cooks opened most oysters.

b. The cooks opened many oysters.

Outside the guarantee's jurisdiction: downward-entailing quantifiers. Let us now turn to downward-entailing quantifiers, which are not covered by the guarantee above. Empirically, it seems that cumulative readings with downward-entailing quantifiers don't require participation from the subject. It is somewhat obvious for (65a), as these inferences would contradict the literal meaning.

(65) a. The 3 cooks opened no oysters.
$\not\rightsquigarrow$ *Cook 1 opened an oyster*
b. Michael and LaToya (together) washed fewer than 3 cars. (Bayer, 2013, p. 198)
$\overset{?}{\rightsquigarrow}$ *Michael washed a car.*

In the account, participation inferences are a form of distributive implicatures. As predicted, these don't seem give rise to distributive implicatures either:

(66) a. No ambassador speaks Arabic, English or Mandarin.
$\not\rightsquigarrow$ *one ambassador speaks Arabic*
b. Less than 10 ambassadors speak Arabic, English or Mandarin.
$\overset{?}{\rightsquigarrow}$ *one ambassador speaks Arabic*

We could stop here and assert that whatever accounts for the absence of distributive implicatures would explain the absence of participation inferences.

For concreteness though, let us try to explain how these facts are derived. For *no* in (67a), absence of participation inferences follows because recursive exhaustification is sensitive to logical contradiction: because the prejacent's meaning in (67b) entails all of the alternatives in (67c) and no exclusion is possible.

(67) **Underlying meaning:**
Susan, Walter and Adrian opened no oysters
a. **Alternatives:**
- Susan and Walter opened no oysters
$\rightsquigarrow \neg\exists x \in \mathsf{oyster}, \exists y \prec \mathsf{Susan} + \mathsf{Walter}, \mathsf{opened}(x)(y)$
- ...

For quantifiers like *less than 10*, a discussion of alternatives are necessary. Unlike upward-entailing quantifiers, downward-entailing quantifiers like "*less than 3 oysters*" can't have "*some oysters*" as their sole alternative. Otherwise, it would be possible to negate the latter and strengthen "*less than 3 oysters*" to "*no oysters*", even in non-cumulative sentences like (68).

(68) Less than 3 oysters are in the box.

This problem is independent from cumulativity. There are essentially two solutions: one could assume that *some* is not an alternative to *fewer than three* or one could assume that *some* is not the *only* alternative to *fewer than 3*. For simplicity, I take the former route: I assume more generally that quantifiers can only have quantifiers of the same monotonicity as alternatives. This is probably an oversimplification[23] but I believe the lack of participation inference will be predicted under a more sophisticated set of assumptions.

If *some* is not an alternative to *less than 3 cars*, then all alternatives, when exhaustified as in (69), contradict the prejacent: they can all be negated but they don't yield a stronger meaning.

(69) **Alternatives:**

- Exh (LaToya washed fewer than 3 cars)
  LaToya washed fewer than 3 cars and not (Michael washed fewer than 3 cars)
- Exh (Michael washed fewer than 3 cars.)
  Micheal washed fewer than 3 cars and not (LaToya washed fewer than 3 cars).

In summary, the account predicts a correlation between presence of distributive implicatures and participation inferences. The lack of distributive implicatures requires an independent explanation and I assume the simplest one, namely that quantifiers only have alternatives of the same monotonicity.

In short, we have the following assumptions:

Assumptions about alternatives

1. $[\![$the NP$]\!]$ has as alternatives all pluralities X, such that $X \prec [\![$the NP$]\!]$
2. upward-entailing quantifiers have *some* as an alternative.
3. quantifiers' alternatives must be of the same monotonicity.

4.2 Asymmetries in cumulative readings

Asymmetries with *every*: the data A major puzzle connected to the cumulative readings of *every* is the presence of asymmetries. Indeed, the cumulative reading of *every* does not obtain when *every* is in the subject position. (70a) is an example: this sentence only has a doubly-distributive reading in which all oysters were opened by each cook (thus implausibly implying that oysters were somehow resealed). Under a cumulative reading, (70a) should be as natural as the cumulative reading of (70b).

(70) a. Every cook opened the 10 oysters. (# cumulative, ✓ doubly-distributive)
b. The 10 cooks opened every oyster. (✓ cumulative,✓ doubly-distributive)

Note that the doubly-distributive readings can also surface with object *every* if we make it more plausible, as in (71). By contrast, even in contexts favoring a cumulative reading (as in scenarios involving oyster-opening), (70a) cannot receive that reading.

(71) The 3 fieldworkers learned every word in the vocab list.

[23] The question of the alternatives to *fewer than n* is complex. First, it is likely that *fewer than n* has other numbers as alternatives (e.g. *fewer than m*). But on their own, these alternatives generate unattested implicatures (*fewer than n but no fewer than n-1*) (Fox and Hackl, 2006). To avoid implicatures, one may invoke alternatives of the form *more than n* (Mayr, 2013). These alternatives create symmetries which make exhaustification vacuous. It may make the set of alternatives too symmetrical: *fewer than 3* has an existence implicature.

To my knowledge, Kratzer (2003) was the first to explicitly note this asymmetry. It is also discussed in Champollion (2010); Ferreira (2005); Haslinger and Schmitt (2018); Ivlieva (2013). Although Kratzer (2003) initially described the asymmetry as an asymmetry in thematic positions, Champollion (2010); Zweig (2008) shows that the asymmetry is one of c-command as expressed in the generalization below:

Generalization

A cumulative reading between *every* and a plural-referring expression is only available when *every* is c-commanded by the plural-referring expression's base position.

Can this generalization be captured in the current analysis?

Analysis. Within the theory of this chapter, the underlying meanings of sentences with *every* in subject position and the sentences with *every* in object position are parallel: the plural-referring expression, in combination with the verb, receives an existential interpretation, which takes scope under the universal quantifier.

(72) a. Every cook ∃-opened the 3 oysters.
b. **Underlying meaning:**
$\forall y \in \mathsf{cook}, \exists x \prec \iota\mathsf{oysters}, \mathsf{open}(x)(y)$

(73) a. The 3 cooks ∃-opened every oyster.
b. **Underlying meaning:**
$\forall y \in \mathsf{oyster}, \exists x \prec \iota\mathsf{cooks}, \mathsf{open}(y)(x)$

This lack of asymmetry in underlying meanings means that the two sentences will have parallel truth-conditions (*mutatis mutandis*) under negation. This prediction is borne out: as reported in Križ and Chemla (2015), the negation of sentences like (72) has the truth-conditions in (74a). These truth-conditions are the same as the truth-conditions of the negation of (73), which we already discussed in section 2, repeated in (74b).

(74) a. Not every cook opened the 3 oysters.
=*not every cook opened an oyster*
=*some cook opened no oyster*
b. The 3 cooks didn't open every oyster.
=*not every oyster was opened by a cook*
=*some oyster wasn't opened by any cook*

The parallel in underlying meanings suggest that any difference between subject *every* sentences and object *every* sentences is due to the way the two sentences are strengthened in positive environments.

Problematically, using the recursive exhaustification at root that used so far, as in (75), delivers the same strengthening for both sentences. (From now on, I will use the symbol $\textsc{Exh}^2$ as an abbreviation for a structure like $\textsc{Exh}_{[b]}\,\textsc{Exh}_{[a]}\ldots$ in both the syntax and the meta-language).

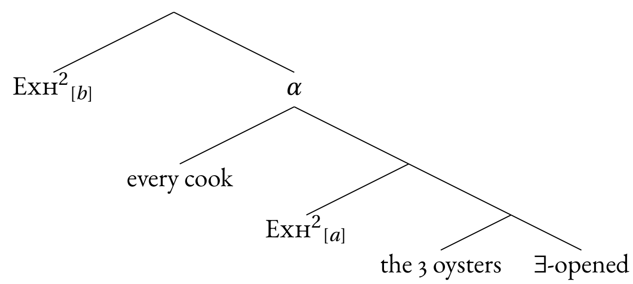
Indeed, these sentences have the same underlying meaning and identical alternatives. Both sentences, as it stands, will receive a cumulative reading, contrary to fact.

(75) a. Exh^2 Every cook ∃-opened the 3 oysters.
⇝ *every cook opened an oyster and every oyster was opened by a cook*.
b. Exh^2 The 3 cooks ∃-opened every oyster.
⇝ *every cook opened an oyster and every oyster was opened by a cook*

The reason for the asymmetry stems, I contend, from the structural difference and the scope of Exh. So far, I have assumed that all exhaustification happens at root. But Exh^2 can also apply between the subject and the object. In fact, I adopt the independently motivated assumption in Magri (2011) that Exh^2 must apply in that position, and more generally wherever it can apply[24]. As we'll see, this assumption will explain the asymmetries observed.

To see this, let's consider the effect of an additional Exh^2 between the subject and the object. As earlier with single Exh operators, I distinguish between two occurrences of Exh^2 with [a] and [b]. When *every* is in subject position, the new Exh^2 is inserted in the scope of *every* and c-commands *the three oysters*, as shown in (76).

(76)



Let's start with $\text{Exh}^2_{[a]}$. In this position, the prejacent of $\text{Exh}^2_{[a]}$ is an existential statement over oysters, with sub-domain alternatives and no intervening quantifiers. As we saw in section 3, this is precisely the configuration in which a Free Choice-like inference is generated. Concretely, the existential whose domain is set by *the 3 oysters* will be strengthened to a universal. The resulting VP denotes the set of entities who opened every oyster ; in combination with *every cook*, α expresses doubly distributive truth-conditions: *every cook opened every oyster*.

(77)
$$
\begin{aligned}
[\![\alpha]\!] &= \forall y \in \mathsf{cook},\ \text{Exh}^2(\exists x \prec \mathsf{oyster}_1 + \mathsf{oyster}_2 + \mathsf{oyster}_3,\ \mathsf{opened}(x)(y)) \\
&\qquad\qquad\qquad\quad \textbf{alts: } \exists y \prec \mathsf{oyster}_1 + \mathsf{oyster}_2,\ \mathsf{opened}(x)(y)), \ldots \\
&= \forall x \in \mathsf{cook}, \forall y \prec \mathsf{oyster}_1 + \mathsf{oyster}_2 + \mathsf{oyster}_3,\ \mathsf{opened}(x)(y)
\end{aligned}
$$

The application of $\text{Exh}^2_{[b]}$ does not change the truth-conditions. Indeed, all the alternatives to α (e.g. (78a) and (78b) among many others) are either entailed by the doubly distributive reading (e.g. (78a)) or contradict it (e.g. (78b)). This situation makes an application of (two) Exh vacuous. In other words, the doubly-distributive meaning is as strong a meaning as one can get through exhaustification.

[24] This also means that there is Exh^2 scoping only over the verb, to the exclusion of the subject and the object. However, this Exh^2 is vacuous, since the verb *open* does not have any alternatives.

(78) a. some cook $\text{Exh}^2{}_{[a]}$ opened the 3 oysters.
≈ *some cook opened oyster 1, 2 and oyster 3*

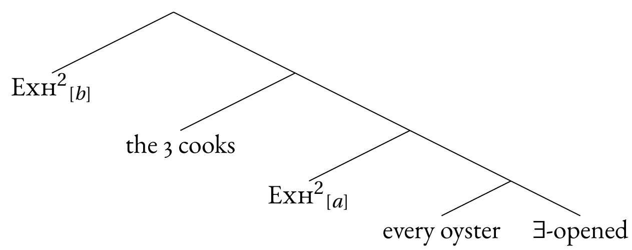
b. every cook $\text{Exh}^2{}_{[a]}$ ∃-opened oyster 1 and oyster 2.
≈ *every cook opened both oyster 1 and 2 but not oyster 3*

All in all, the sentence with subject *every* receives a doubly-distributive interpretation as needed.

By contrast, assuming an embedded Exh in the object-*every* sentence like (75b) we studied, does not prevent a cumulative reading from arising. The structure is as in (79a). There, unlike (77), the set of alternatives to the embedded $\text{Exh}^2_{[a]}$ does not contain alternatives where the existential *open* quantifies over a sub-domain. Such alternatives are obtained by substituting *the 3 cooks*, but the latter is not part of the prejacent of Exh. The alternatives to the prejacent of $\text{Exh}_{[a]}$ are, in sum, only the alternatives to *every*, i.e. *some*.

With only *some* as an alternative, the embedded Exh is vacuous, cf (79b).

(79) a.



b. $[\![(79a)]\!] = \iota\text{cooks}\ \lambda Y.\ \text{Exh}^2{}_{[a]}\,(\forall x \in \text{oyster}, \exists x' \prec x, \exists y \prec Y,\ \text{opened}(x')(Y))$

alts: $\exists x \in \text{oyster}, \exists x' \prec x, \exists y \prec Y,\ \text{opened}(x')(Y))$

$= \iota\text{cooks}\ \lambda Y.\ \forall x \in \text{oyster}, \exists x' \prec x, \exists y \prec Y,\ \text{opened}(x)(Y)$

From then on, with the embedded Exh being vacuous, the composition proceeds as if only root $\text{Exh}^2{}_{[b]}$ was present. A structure with root Exh^2, as we saw, is precisely how the cumulative reading is generated.

In sum, the assumption that Exh^2 must apply wherever it can derives different strengthening depending on the structure: doubly-distributive strenghtening when *every* is a subject, cumulative when it is an object. Taking a step back, the meaningful structural difference between subject *every* and object *every* is that in the latter case, the sub-domain alternatives of *the 3 cooks* are only visible after *every* has merged and strengthening can thus happen across *every*.

Adding recursive exhaustification in every position to our list of assumptions, we reach the final list of assumptions:

Assumptions

1. **assumptions about composition**
   - verbs have existential meanings (e.g. ∃-*opened*)
   - recursive Exh in positive environments *in all positions.*.
2. **assumptions about alternatives**
   - ⟦the NP⟧ has as alternatives all pluralities X, such that $X \prec$ ⟦the NP⟧
   - "*every*", *many*, *most* have *some* as an alternative.
   - quantifiers' alternatives must be of the same monotonicity.

4.3 Ordinary cumulative readings

With these assumptions, we are ready to come back to ordinary cumulative readings, like (80). Ordinary cumulative readings do not raise particular issues for the theory, but the full set of assumptions made so far is necessary to account for it.

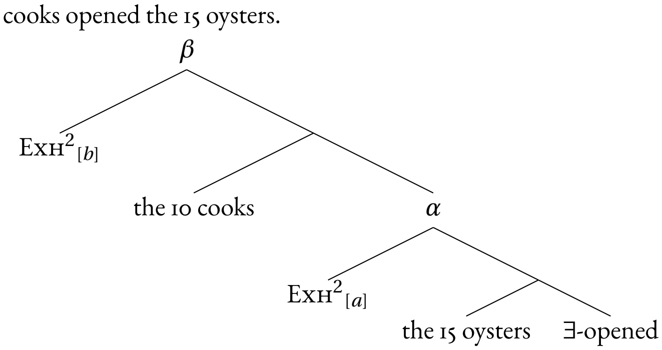
(80) a. The 10 cooks opened the 15 oysters.
  b. The 10 cooks didn't open the 15 oysters.

The negative case in (80b) is the simplest. In the scope of negation, no strengthening through Exh^2 occurs ; by the existential meaning of *open*, both plural-referring expressions are interpreted as existentials in the scope of negation. The predicted meaning matches the attested meaning: *no cook opened any oyster*.

(81) a. not [the 10 cooks ∃-opened the 15 oysters]
  b. ⟦(81a)⟧ $= \neg\exists x \prec \iota\text{cooks}, \exists y \prec \iota\text{oysters}, \text{opened}(y)(x)$
  ⇜⇝ *no cook opened any oyster*

In the positive case of (82a), exhaustification is active. As seen in the last section, we need to include one Exh^2 in all positions.

(82) a. The 10 cooks opened the 15 oysters.
  b.



The computation is arduous, but we can develop a simple intuition for how it will run. In the embedded position α, Exh$^2{}_{[a]}$ operates over the sub-domain alternatives of the existential whose domain is *the oysters*, cf (83). Because there is no intervening quantifier, this existential will be strengthened to a universal (i.e. a Free Choice-like inference), cf (83a).

(83) $[\![\alpha]\!] = \lambda X.\ \text{Exh}^2{}_{[a]}(\exists y \prec \iota\text{oysters}, \exists x \prec X,\ \text{opened}(y)(x))$
alts: $\exists y \prec \text{oyster}_1 + \text{oyster}_2, \exists x \prec X,\ \text{opened}(y)(x)), \ldots$
$= \lambda X.\ \forall y \prec \iota\text{oysters}, \exists x \prec X,\ \text{opened}(y)(x)$
$\leftrightarrow$ *every oyster was opened by one of X*

In the root position β, $\text{Exh}^2{}_{[b]}$ operates over the sub-domain alternatives of *the 10 cooks*[25]. Here however, the existential represented by *the cooks* finds itself in the scope of the universal corresponding to *the 15 oysters* which was created by the first strengthening. The situation is entirely parallel to the case of cumulative readings of *every* ; Exh^2 will generate a distributive-like implicature. Together with the prejacent, this will create the cumulative reading.

(84) a. $[\![\beta]\!] = \text{Exh}^2{}_{[b]}(\forall y \prec \iota\text{oysters}, \exists x \prec \iota\text{cooks opened}(y)(x))$
alts: $\exists y \in [\![\text{oyster}]\!], \exists x \prec \text{Joana} + \text{Marius},\ \text{opened}(y)(x)), \ldots$
$= \forall y \prec \iota\text{oysters}, \exists x \prec \iota\text{cooks},\ \text{opened}(y)(x)$
$\land \forall x \prec \iota\text{cooks}, \exists y \prec \iota\text{oysters},\ \text{opened}(y)(x)$
$\leftrightarrow$ *cumulative reading*

All in all, the computation raises no particular issue. The object is first strengthened to a universal meaning ; from then on, the situation is entirely parallel to the case of cumulative readings of *every*.

4.4 Bare numerals

In the classical literature on cumulativity (Landman, 2000; Scha, 1984; Schein, 1993; Sternefeld, 1998), cumulative examples are usually presented with bare numerals, as in (85).

(85) a. Three cooks opened fifteen oysters.
b. Three cooks opened every oyster.

The presentation has so far privileged referential expressions, like definites. One reason is that using referential expressions removes one unnecessary layer of quantification. Another reason is that bare numerals also involve an extra complication: they don't give rise to weak meanings under negation (Kriz, 2015) ; in other words, they are not homogeneous.

(86) a. Three cooks didn't open every oyster.
b. **Truth-conditions:**
It's not the case that there are three cooks such that all oysters were opened by one of them and each of them opened an oyster

This is not limited to cumulative sentences. Weak meanings are not observed in simple intransitive sentences either. The truth-conditions of (87a) are the exact complement of the truth-conditions of (87b).

(87) a. Three people smiled.
b. It's not the case that three people smiled.

[25] In addition to the sub-domain alternatives of *the 15 oysters*.

This is a *prima facie* problem for the present theory[26], which insists that cumulative readings arise as strenghtened weak existential readings. Where no weak readings are observed, no cumulative readings should arise.

Importantly though, not observing weak readings at the level of the sentence does not mean that weak existential meanings didn't appear at some point during the composition. There is evidence that weak meanings still occur in sentences with bare numerals. This can be seen when the numerals take scope over negation, as in (88b). If *smile* intrinsically requires all participants to smile, (88) should mean (88b). But it means (88c), as expected if *smile* is ∃-smile.

(88) Three people weren't smiling.
  a. **LF:** three people λX. not [X smile]
  b. three people are such that not all of them smiled
  c. three people are such that none of them smiled

To account for (88), we must thus assume that the numeral has the effect of strengthening the existential meaning to a universal, even in negative environments. In a slogan, it performs *homogeneity removal* (Kriz, 2015).

Within the theory of Bar-Lev (2018b) which we build on, the effect of homogeneity removal is obtained by assuming that bare numerals must co-occur with $\textsc{Exh}^2$ and that the subdomain alternatives cannot be "*pruned*" in its presence. The origin of a constraint forbidding the pruning of certain alternative is a puzzle ; however, we note the idea that certain operators require mandatory exhaustification has already been proposed for exceptives (Hirsch, 2016), boundary adverbials (Iatridou and Zeijlstra, 2021) and NPIs (Chierchia, 2013). So it is not a puzzle specific to this theory *per se*.

Concretely, I assume that a $\textsc{Exh}^2$ must occur under the abstract created by the numeral, as in (89b). This is also required when it is present under negation as in (90).

(89) a. Three people smiled.
  b. [three people] λY. $\textsc{Exh}^2$ Y smiled

(90) a. Three people didn't smile.
  b. **LF:** not [three people] λY. $\textsc{Exh}^2$ Y smiled

A simplification made here is to ignore the *exact* implicatures of numerals. Giving an account of such implicatures in cumulative contexts is tantamount to an account of cumulative readings of *modified* numerals (Brasoveanu, 2013; Landman, 2000), a challenging problem I set aside in fn. 5. Formally, this means that we do not consider *four oysters* and other numerical phrases to be alternatives to *three oysters*. I nevertheless assume that numerals (*four oysters*) have existentials *some oysters* as alternatives. In other words, I treat the numeral like all other quantifiers discussed in this work, like *every*.

[26] I thank two reviewers for pressing me to discuss this point.

This remark aside, the stipulations made so far to explain the effect of numerals in simple intransitive sentences, like (88) and (87b), translate to an account for cumulative readings of numerals. Let us consider a simplified cumulative sentence with one numeral, as in (91a), represented by the structure in (91b)[27].

(91) a. The ten cooks opened three oysters
b. Exh^2 [the ten cooks] [three oysters] λY. [Exh^2 ∃-opened $Y]_\alpha$

Assuming that the trace Y has its sub-pluralities as alternatives, the meaning of the constituent α is parallel to the meaning of e.g. "*opened the fifteen oysters*" in (82b) of the previous sections. Namely, it is an et predicate true of X if every member of Y (a free variable) was opened by one of X.

(92) $[\![\alpha]\!] = \lambda X.\ \text{Exh}^2(\exists y \prec Y, \exists x \prec X,\ \mathsf{opened}(y)(x))$
$= \lambda X.\ \forall y \prec Y, \exists x \prec X,\ \mathsf{opened}(y)(x)$
$\leftrightarrow$ *every member of Y was opened by one of X*

The constituent α then combines with the bare numeral and the definite, yielding (93).

(93) $[\![$[the ten cooks] [three oysters] $\lambda Y.\ \alpha]\!]$
$= \exists Y \in \mathsf{oysters}, |Y| = 3 \wedge \forall y \prec Y, \exists x \prec \iota\mathsf{cooks},\ \mathsf{opened}(y)(x)$
$\leftrightarrow$ *every member of Y was opened by one of X*

Recursive exhaustification applies to the meaning in (93). Here, we are within the domain of the "*UE entails participation*" guarantee. The under-braced part of the sentence, corresponding to *three oysters*, is an upward quantifier with *some* as an alternative (*three oysters* was stipulated ealier to have *some oysters* as its alternative). *the ten cooks* is a plural-referring expression with sub-pluralities as alternatives. By the results of section 4.1, we know that the matrix Exh^2 will deliver participation inferences, as in (94). The truth-conditions are adequate.

(94) $\text{Exh}^2 \underbrace{\exists Y \in \mathsf{oysters}, |Y| = 3 \wedge \forall y \prec Y,}_{\mathcal{Q}y,} \exists x \prec \iota\mathsf{cooks},\ \mathsf{opened}(y)(x)$
By "*UE entails participation*",
$= \exists Y \in \mathsf{oysters}, |Y| = 3 \wedge \forall y \prec Y, \exists x \prec \iota\mathsf{cooks},\ \mathsf{opened}(y)(x)$
$\wedge \forall x \prec \iota\mathsf{cooks}, \exists x \in \mathsf{oyster},\ \mathsf{opened}(y)(x)$
three oysters were opened by some cook and every cook opened an oyster

The case of cumulative sentences with two numerals is only marginally different from the one studied here. By stipulation, *two cooks* must co-occur with Exh^2. The resulting structure is as given in (95):

(95) a. Two cooks opened four oysters.
b. **LF:** two cooks λX. [Exh^2 X [three oysters] λY. Exh^2 ∃-opened $Y]_\beta$

[27] A reviewer wonders whether the necessity of QR with numerals might imply that cumulativity with numerals would be more restricted than with definites (e.g. because QR is subject to island constraints). It is not clear to me that this is so. The restrictions on QR only concern the scope of the indefinites ; as we'll see, it is mostly through traces that the cumulative relation is established.

The bracketed constituent β is structurally parallel to the sentence with just one numeral investigated in (93) earlier. We thus know its truth-conditional meaning to be as in (96a). Applying the meaning of the numeral to it, yields the desired truth-conditions in (96b)[28].

(96) a. $[\![\beta]\!] = \exists Y \in \mathsf{oysters}, |Y| = 3 \wedge \forall y \prec Y, \exists x \prec X, \mathsf{opened}(y)(x)$
$\wedge \forall x \prec \iota\mathsf{cooks}, \exists x \in \mathsf{oyster}, \mathsf{opened}(y)(x)$

b. $[\![(95b)]\!] = \exists X \in \mathsf{cooks}, |x| = 2 \wedge \exists Y \in \mathsf{oysters}, |Y| = 3 \wedge \forall y \prec Y, \exists x \prec X, \mathsf{opened}(y)(x)$
$\wedge \forall x \prec X, \exists x \in \mathsf{oyster}, \mathsf{opened}(y)(x)$ *There are two cooks such that (a) three oysters were opened some of them, (b) all of them opened an oyster.*

Finally, let's look at asymmetries with bare numerals. As with definites, (97a) exhibits a cumulative reading that (97b) does not.

(97) a. Three cooks opened every oyster.

b. Every cook opened three oysters.

This asymmetry is explained in exactly the same way as the asymmetry observed with definites. The sentences in (97) requires more LF raising than ordinary cumulative sentences, owing to the special configuration I assumed bare numerals must occur in. (98a) generates a cumulative reading ; at a glance, *every oyster* combines with the verb before the subject's trace Y. This means that participation inferences will be computed over a constituent whose meaning is "*every oyster was opened by some of* Y", yielding a cumulative reading. In (97b), the trace of the object is strengthened prior to combination with *every*. The VP as a whole must has a strong universal meaning: "*Y opened all of four oysters*".

(98) a. [three cooks] λY. EXH^2 Y [every oyster] λZ. EXH^2 ∃-opened Z.

b. [every cook] λY. EXH^2 Y [four oysters] λZ. EXH^2 ∃-opened Z

To sum up, it's always the fact that the trace of the definite/bare numerals can stand higher than *every* at LF, which permits cumulative reading. When this condition is not met, the definite/bare numeral is strengthened to a maximal reading before combining with *every*.

4.5 Summary

In this section, four extensions to the basic theory of cumulative readings of *every*: cumulative readings of non-*every* quantifiers, cumulative asymmetries, ordinary cumulative sentences, bare numeral cumulative sentences. The final account makes the following assumptions about exhaustification and alternatives:

[28] As announced earlier, these truth-conditions allow more cooks to have opened oysters and more oysters to have been opened.

Assumptions

1. **assumptions about composition**
   - verbs have existential meanings (e.g. ∃-*opened*)
   - recursive Exh in positive environments *in all positions.*.
2. **assumptions about alternatives**
   - $[\![\text{the NP}]\!]$ has as alternatives all pluralities X, such that $X \prec [\![\text{the NP}]\!]$
   - There is no ∀-VP alternative to ∃-VP.
   - "*every*", *many*, *most* have *some* as an alternative.
   - quantifiers' alternatives must be of the same monotonicity.

5 Comparison to previous literature and broader issues

This section opens a broader discussion on cumulativity. In the first section, I discuss approaches from previous literature and compare them to the current system. In the second section, I discuss issues arising when trying to incorporate collective action into the current analysis.

5.1 Previous literature

In this section, I review several alternative solutions to the problem raised by cumulative readings of *every* as found in previous literature. An immediate point of comparison is that many previous solutions don't discuss negative sentences and therefore make incorrect predictions as is. We'll focus on solutions that do or can incorporate homogeneity and cumulativity. The general edge of the present theory with respect to these homogeneity-conscious theories is that its theory of quantification is entirely standard. In the present account, only assumptions about alternatives need to be made but these are independently evidenced by studying distributive implicatures, as seen in sec. 4.1. In other theories, the semantics of quantifiers need to be adapted. In the case of event semantics, the adaptation creates conflicts with other aspects of the theory. The plural projection framework, on the other hand, can address all the data points. I thus makes an empirically matched competitor ; however, unlike the present theory, the enrichments to the semantics it requires don't have independent motivation.

5.1.1 Event semantics

Starting with Schein (1993), cumulative readings of *every* have often been accounted for in terms of Neo-Davidsonian event semantics (Champollion, 2016; Ferreira, 2005; Ivlieva, 2013; Kratzer, 2000; Schein, 1993). In such accounts, the denotation of *every* is typically changed to an event-sensitive denotation, allowing it to interact with event composition.

(99) illustrates a prototypical derivation, loosely following Champollion (2016): *every oyster* combines with the VP to form a predicate true of events which are sums of oyster-openings, in which all oysters were opened. The subject "*the cooks*" is asserted to be the agent of one such event.

(99) a. The cooks opened every oyster.

b. **LF:** $\exists e$. [the cooks AGENT] [every oyster] λx. opened [x THEME]

c. $[\![\text{every oyster}]\!] = \lambda p.\lambda e.\ (e, \bigoplus \text{oyster}) \in {}^{*} \left[\lambda x.\lambda e.\ p(e) \wedge \textsc{Theme}(e) = x \wedge \mathsf{oyster}(x)\right]$

$[\![\text{VP}]\!] = \lambda e.\ (e, \bigoplus \text{oyster}) \in {}^{*} \left[\lambda x.\lambda e.\ \mathsf{open}(e) \wedge \textsc{Theme}(e) = x \wedge \mathsf{oyster}(x)\right]$

$=$ *e is a sum of oyster openings in which all oysters were opened*

d. $[\![(99\text{a})]\!] = \exists e, \textsc{Agent}(e) = \iota\mathsf{cooks} \wedge (e, \bigoplus \text{oyster}) \in {}^{*} \left[\lambda x.\lambda e.\ \mathsf{open}(e) \wedge \textsc{Theme}(e) = x \wedge \mathsf{oyster}(x)\right]$

The resulting truth-conditions correctly assert that in one or more openings, the cooks did open the oysters, which is the cumulative reading. In most traditional event analyses, homogeneity is not accounted for and negative sentences are not discussed. Chatain (2021a) provides one way homogeneity, cast using Kriz (2015)'s theory of homogeneity, can be incorporated into an event semantics. He proposes that thematic roles receive homogeneous readings[29].

Even then, there are drawbacks to an event approach. For the event analysis to work, the denotation of *every* need to be adapted to event semantics in order to deliver the correct reading. The semantics of other quantifiers which partake in cumulative readings, like non-partitive *most* or *fewer than three*, similarly needs to be adapted. The need for adapting quantifier denotations creates issues for the event solution. First, independently from any considerations of cumulativity, the theory of quantification in event semantics has problems that do not arise in a more traditional generalized quantifier theory. Downward-entailing quantifiers, for instance, "*require particular care*", as Kratzer (2003) puts it, because naive adaptations of quantifiers into event semantics generate unattested readings. To be sure, several solutions to the problem of quantification in event semantics have been proposed (Champollion, 2014a; de Groote and Winter, 2015; Krifka, 1989; Winter and Zwarts, 2011). For instance, Champollion (2014a); de Groote and Winter (2015); Winter and Zwarts (2011) each propose a system in which sentences are composed in such a way that quantifiers effectively always take high scope with respect to the event existential.

This may seem orthogonal to the problem of cumulative readings of *every* but it isn't. As sketched in (99), the analysis of cumulative readings requires on the other hand that quantifiers take a *low* scope with respect to the event quantification. In short, solutions to the problem of quantification in event semantics interact poorly with solutions to the problem of cumulative readings of quantifiers. This is already noted: Champollion (2014b) observes that, at least provisionally, the theory of quantification in Champollion (2014a) cannot be reconciled with the theory of *every* in Champollion (2016), which is used for cumulative readings.

The force of the present account is that it latches onto a standard theory of quantification ; the problems of quantification in event semantics do not arise. As we saw, the account readily extends to other quantifiers like *fewer than three*, as seen in section 4.1.

5.1.2 Plural projection framework

In Haslinger and Schmitt (2018), the plural projection framework of Schmitt (2013) is applied to cumulative readings of *every*. The plural projection framework relies on two premises: 1)

[29] There are reasons to believe that this is the wrong locus of homogeneity. As discussed in fn. 14, some verbs resist homogeneity. The approach in Chatain (2021a) derives homogeneity for all predicates, through the assumption that all thematic roles carry homogeneity and therefore can't directly accommodate that fact.

pluralities are cross-categorial: in addition to pluralities of individuals, there are pluralities of propositions, predicates, etc., 2) pluralities combine cumulatively.

To account for cumulative readings of *every*, Haslinger and Schmitt (2018) assume that *every oyster* combines with the relation denoted by the verb to form a plural predicate, as in (100a). More accurately, it denotes a singleton containing a plural predicate: to deal with indefinites, the system also assumes a form of Alternative Semantics (Kratzer and Shimoyama, 2017). This plural predicate may compose *cumulatively* with the plural subject to form the cumulative reading of the sentence.

(100) a. The cooks opened every oyster.
b. ⟦opened every oyster⟧ = $\{\mathsf{open}(\mathsf{oyster}_1) \oplus \ldots \oplus \mathsf{open}(\mathsf{oyster}_n)\}$

The composition of the VP in an ordinary cumulative sentence proceeds differently but lands the same result.

(101) a. The cooks opened every oyster.
b. ⟦opened the oysters⟧ = $\{\mathsf{open}\left(\mathsf{oyster}_1 \oplus \ldots \oplus \mathsf{oyster}_n\right)\}$
$= \{\text{open}(\mathsf{oyster}_1) \oplus \ldots \oplus \text{open}(\mathsf{oyster}_n)\}$

This parallel between ordinary cumulative sentences and cumulative sentences with *every* makes accounting for the difference in their homogeneity properties challenging. Indeed, as discussed in Chatain (2021a), a natural implementation of homogeneity in the plural projection framework (as in e.g. Schmitt (2017)) will make the two sentences exactly equivalent under negation, contrary to what we observe:

(102) a. The cooks didn't open every oyster.
b. The cooks didn't open the oysters.

Haslinger (2021)'s proposal[30], embedded in the same framework, offers a proposal which resolves the problem. The idea is to use a bi-dimensional semantics in addition to the rest of the machinery. She proposes that constituents have two types of denotation: a strict denotation and a tolerant denotation. The strict denotation gives the truth-conditions of a positive sentence and the tolerant denotation gives the falsity-conditions of the negation ; it corresponds to the unstrengthened meaning in the theory presented here. In this theory, *the oysters* and *every oyster* would have the same strict truth-conditions. Their tolerant denotations differ: *the oysters* tolerantly denotes something like the indefinite *some oysters*. The tolerant denotation of *every oyster* is equal to its strict denotation, indicating no truth-value gap.

[30] I focus on cumulative readings of quantifiers but Haslinger (2021)'s proposal is richer. Missing from my discussion is her sophisticated account of non-maximality. In addition, Haslinger makes the intriguing claim that the cumulative and distributive construals are not the result of ambiguity but under-specification. This claim would affect the proposal here and any proposal in plural semantics more generally but I will not weigh on it here: as Bar-Lev (2020) goes to show, under-specified readings may depend on the predicate at hand, making matters considerably more difficult than appears at first blush.

(103) a. Strict denotations (written $[\![\dots]\!]_+$):
$[\![\text{the oysters}]\!]_+ = \mathsf{oyster}_1 \oplus \dots \oplus \mathsf{oyster}_n$
$[\![\text{every oyster}]\!]_+ = \mathsf{oyster}_1 \oplus \dots \oplus \mathsf{oyster}_n$
b. Tolerant denotations (written $[\![\dots]\!]_?$):
$[\![\text{the oysters}]\!]_? = \{X \mid X \prec \mathsf{oyster}_1 \oplus \dots \oplus \mathsf{oyster}_n\}$
$[\![\text{every oyster}]\!]_? = \mathsf{oyster}_1 \oplus \dots \oplus \mathsf{oyster}_n$

Glossing over the composition, this difference will entail that (104) and (105) will have the same strict truth-conditions but different tolerant denotations. Since tolerant denotations drive how negative sentences are understood by default in Haslinger (2021)'s system, the system correctly predicts the expected discrepancy in truth-conditions between the two sentences.

(104) The cooks opened the oysters
a. **strict:** the cooks cumulatively opened the oysters.
b. **tolerant:** some cooks cumulatively opened some oysters.

(105) The cooks opened every oyster
a. **strict:** the cooks cumulatively opened the oysters.
b. **tolerant:** some cooks cumulatively opened the oysters.

With this enrichment, the two theories become matched in their predictions. There remain some important points of comparison. First, since the tolerant and the strict denotation are defined separately in Haslinger (2021), nothing dictates that they should bear any relationship to each other ; if nothing is said, an item with *some* as its tolerant denotation might well have *most* as its strict denotation. This pattern of homogeneity is never observed as far as I know. By contrast, the theory presented in this paper is less expressive. It relates its weak meanings to its strong meanings, via exhaustification. It is not possible to change the weak meaning without altering the strong meaning. Similarly, altering the exhaustification procedure alters strong meanings and implicatures across the board. To put it differently, the theory of this work comes with an implicit generalization on what the shape of the gap can be (i.e. it is similar to a Free Choice/distributive implicature gap).

A second point of comparison concerns generalized quantifiers. It is clear how certain quantifiers can be lifted into Haslinger (2021)'s semantics. For upward monotone plural quantifiers, it suffices to set their strict and tolerant denotations to the set of their witnesses. This is less clear for downward-entailing quantifiers. Haslinger and Schmitt (2020) offers an idea for a solution but this proposal requires yet another enrichment to the semantics. By contrast, in the theory presented here, accommodating new quantifiers only requires stipulating their set of alternatives, a task partially constrained by the implicatures they independently give rise to. The composition can be left as is. Of course, the price paid by this theory to achieve this result is a more complex syntax, using obligatory recursive exhaustification. However, as argued, this syntax is motivated by reference to other phenomena, e.g. blind implicatures (Magri, 2014) and Free Choice phenomena (Fox, 2007).

These arguments are far from showing that the plural projection approach is nonviable. But they are sufficient to establish the approach in this work as an attractive competitor.

5.2 Collective interpretations

So far, the cumulative examples were given a paraphrase as in (106b).

(106) a. The ten cooks opened every oyster.
  b. **Truth-conditions:**
  Every cook opened an oyster.
  Every oyster was opened by a cook.

Switching to predicates with a more plausible collective interpretation, as in (107), a parallel paraphrase in (107b) comes out too strong: it implies that every jigsaw puzzle was completed *individually*. A more adequate paraphrase would be (107c) where it is merely implied that every player participated in the completion of a jigsaw puzzle.

(107) a. The ten players completed every jigsaw puzzle.
  b. **Incorrect truth-conditions:**
  Every jigsaw puzzle was completed by a player.
  Every player completed a jigsaw puzzle.
  c. **Correct truth-conditions:**
  Every jigsaw puzzle was completed by a group of players.
  Every player was part of a group of players that completed a jigsaw puzzle.

The theory so far predicts the incorrect (107b). In the sequel, I will try to discuss a potential path to solving this issue, ultimately leaving it open. This solution won't derive the correct (107c), but the truth-conditions derived are equivalent to those that Vaillette (1998) and Harnish (1976) proposed underlie collective predication.

Starting small, we attempt to derive the homogeneity and truth-conditions of the sentences in (108). As discussed in Kriz (2015), collective predicates bring their own form of homogeneity. The negative sentence in (108b) denies the existence of any plurality merely overlapping with the players who completed Jigsaw 1. Overlapping is understood in its loosest sense here, counting plurality containing or contained in the players. This is dubbed *sidewards* homogeneity in Kriz (2015).

(108) a. The players completed Jigsaw 1.
  b. The players didn't complete Jigsaw 1.
  ≈ *no player was part of a group that completed jigsaw 1.*

Following this paper's strategy, we take (108b) to represent (the negation of) the underlying truth-conditions of (108a). We thus posit the verbal denotation for *complete* in (109)[31].

(109) $[\![\exists\text{-complete}]\!] = \lambda y.\lambda X.\exists x < X, \exists X' > x, [\![\text{complete}]\!](y)(X')$
  some group overlapping with X completed y

[31] The verb *complete* is distributive in its object position (i.e. *Joshua completed the two jigsaw puzzles* is equivalent to the conjunction of *Joshua completing Jigsaw puzzle 1* and *Joshua completing Jigsaw puzzle 2*). The general case of a verb collective in both its arguments, like the transitive *gather* is $\lambda Y.\lambda X.\exists x < X, \exists X' > x, \exists y < Y, \exists Y' > y, [\![\text{gather}]\!](Y')(X')$. We set such cases aside for simplicity.

By our assumptions, (108a) will be parsed as (110a) (ignoring vacuous embedded exhaustification). The prejacent of $\textsc{Exh}^2$ will receive the truth-conditions in (110b). Recursive exhaustification will generate a Free Choice inference, turning the existential quantification over atomic parts of the players to a universal one. The resulting meaning in (110c) asserts that every player was part of a group who completed Jigsaw 1.

(110) a. $\textsc{Exh}^2$ [the players completed Jigsaw 1]$_\alpha$
b. $[\![\alpha]\!] = \exists x \prec \iota\text{players}, \exists X' \succ x, [\![\text{complete}]\!]\,(\text{jigsaw}_1)(X')$
c. $[\![\textsc{Exh}^2 \alpha]\!] = \underline{\forall x \prec \iota\text{players},}\exists X' \succ x, [\![\text{complete}]\!]\,(\text{jigsaw}_1)(X')$
d.

These truth-conditions are weaker than the expected truth-conditions ; while they guarantee that all players took part in the completion of the puzzle (exhaustive participation), they do not exclude that the players did so in collaboration with non-players[32]. This is admittedly not the intuition speakers access from (108a). Yet, Vaillette (1998), following Harnish (1976), argues that these are indeed the correct underlying truth-conditions. One argument for this view is that, if external collaboration were ruled out in the underlying semantics, the semantics of modifiers like *alone* as in (111a) would be vacuous in the positive form.

(111) a. The players completed Jigsaw Puzzle 1 alone.
b. The players didn't complete Jigsaw Puzzle 1 alone.

The difficulty resides in spelling out exactly how the truth-conditions get (further) reinforced to the observed truth-conditions. This might involve yet another implicature process. This raises many issues which we don't solve here: how does this implicature process work? which alternatives does it operate on? how does it interact with the implicature generated here? While there remains much to spell out, this view has the advantage of (i) shedding light on the phenomenon of *sidewards homogeneity*, (ii) providing an understanding of the semantics of modifiers like *alone*.

Finally, these simple truth-conditions extend to the cumulative case. We start from the meaning of the prejacent (112b). By exhaustification, the prejacent will obtain the meaning in (112c), similarly to a distributive implicature.

(112) a. The players completed every jigsaw puzzle.
b. $[\![\alpha]\!] = \forall j \in \text{jigsaw-puzzle}, \exists x \prec \iota\text{players}, \exists X' \succ x, [\![\text{complete}]\!]\,(j)(X')$
c. $[\![\textsc{Exh}^2 \alpha]\!] =$
$\forall j \in \text{jigsaw-puzzle}, \exists x \prec \iota\text{players}, \exists X' \succ x, [\![\text{complete}]\!]\,(j)(X')$
$\wedge \forall x \prec \iota\text{players}, \exists j \in \text{jigsaw-puzzle}, \exists X' \succ x, [\![\text{complete}]\!]\,(j)(X')$
every puzzle was completed by a group containing a player
and every player was part of a group that completed the puzzle

The truth-conditions does assert that every player was part of a group who completed a puzzle and that every puzzle was completed by a group containing a player.

[32] These truth-conditions also do not require that the players complete the work as a single team, but that allow for the puzzle to have been completed multiple times by multiple teams of players working independently from one another. In that, we side with the conventional wisdom from Kratzer (2003); Schein (1993), although it is not unchallenged (Bayer, 2013).

Like the non-cumulative case, this is weaker than the intuitive reading but allows for players to collaborate with non-players for puzzle completion. Following Vaillette (2001) and Harnish (1976), I speculate that the correct truth-conditions are derived from these truth-conditions via another implicature but I leave it open exactly how this is achieved.

This lacuna also affects other theories, in particular (Haslinger and Schmitt, 2018, fn. 9). As far as I can tell, only the event semantics fares well on these cases. However, the latter, as discussed in section 5.1.1, has problems of its own regarding quantification, which the current theory does not have.

Conclusion

This paper proposed a new theory of cumulative readings of *every* and quantifiers in object position. This theory of cumulativity is special in that it does justice to the homogeneity properties of cumulative readings and gives an account of the truth-conditions of negative sentences, not frequently addressed in previous approaches.

The assumptions are repeated below. It builds at its core an existential meaning in the denotation of the verb. This existential meaning gives rise to strong readings under negation (*homogeneity*) and permits cumulative readings of *every* to arise. The existential meaning alone is sufficient to derive the meaning of negative sentences but fails to account for the participation inferences which arise in positive sentences. I developed an analogy between these inferences and the implicatures of Free Choice/distributive implicatures. I showed that an account in terms of recursive exhaustification accounts for both Free Choice/distributive implicatures and participation inferences in a parallel fashion. The account can be extended to account for the cumulative readings of other quantifiers, and for asymmetries in cumulative readings.

Assumptions

1. verbs have existential meanings (e.g. ∃-*open*).
2. recursive Exh in positive environment *in all positions.*.
3. each sub-plurality is an alternative to a plural-referring expression.
4. existential alternatives to quantifiers.

A limitation of the theory is that it does not account for all cumulative readings of quantifiers. In particular, cumulative readings of modified numerals (Brasoveanu, 2013; Buccola and Spector, 2016; Landman, 2000; Schein, 1993) are not dealt with.

(113) More than 15 children ate less than 8 ice-creams.

It is interesting to note that two theories of such readings start of with an underlying meaning akin to "*some of the children ate some of the ice-creams*". In Brasoveanu (2013), the assertive component of the modified numeral is existential (its post-suppositional component contains the numeric test). In Landman (2000), this corresponds to the SC reading generated below the maximalization operator. It would be interesting to see if and how such existential readings can be connected to the existential readings posited in the current theory. This endeavor is left to future research.

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