Exhaustive readings of indefinites require rich content

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Abstract. A number of environments in natural language, such as *only* or term answers, give rise to exhaustive readings. In a tradition pioneered by Rooth (Rooth 1985), the exhaustive reading in these environments is obtained from a prejacent and alternatives obtained by replacing the focus of the sentence with appropriate replacements. In Rooth (1985) and many subsequent work, the prejacent and the alternatives are taken to be worlds or sets of worlds. This paper shows that exhaustive readings of indefinites raise a challenge for this view. That exhaustive readings of indefinites raise issues is known from earlier works (van Rooij and Schulz 2004). But this paper shows that the challenge is in fact quite general, affecting all accounts, existing or to be invented, which treat the prejacent and its alternatives as propositional items. In particular, it affects more recent accounts of exhaustivity such as Fox (2007), which were claimed to be immune from it. The second contribution of this article is to provide a solution in terms of dynamic approach, which adds to a Roothian approach, just the necessary amount of dynamic semantics necessary to properly solve the challenge of exhaustive readings of indefinites. It is argued that this account compares favorably to previous accounts in terms of enriched content (Bonomi and Casalegno 1993; van Rooij and Schulz 2004).

I. Introduction

Sentences with *only*, as in (2), and term answers to questions, as in (3), differ in meaning from the bare sentence in (1), in that they give rise to *exhaustivity readings*. In its simplest description, the exhaustive reading asserts that nothing relevant is true, beyond what (1) says.

(1) Jane was informed of the incident.

(2) Only JANE was informed of the incident.
→ *Bill wasn't, Sean wasn't*

(3) _ Who was informed of the incident?
_ JANE.
→ *Bill wasn't, Sean wasn't*

In a tradition started by Rooth (1985), exhaustive readings are thought to arise by competition of the bare sentence (hereafter *the prejacent*) element with *alternatives* to that prejacent which replaces the content of the focus with alternative expressions. Formally, the exhaustive reading can be represented as an exhaustification procedure, as in (4), which takes as input the prejacent and alternatives to the prejacent obtained by substituting the focus with other elements.

(4) EXH (Jane was informed, {Bill was informed, Sean was informed})

In (2), the semantics of *only* embodies the procedure itself. In (3), it is controversial whether this procedure describes the product of pragmatic reasoning or is the semantics of a covert operator EXH, whose meaning is akin to *only*.

Setting aside where the procedure lies in the grammar, the focus of this article is to make a claim regarding the exhaustification procedure. The starting point is the known fact that exhaustive readings of indefinites, like (5), where the focus is an indefinite expression, represents a difficult problem for theories of the exhaustification procedure. In the past, such sentences have been used as motivation for new accounts of exhaustification.

(5) This bag only contains PICTURES OF BOB MARLEY.
→ *this bag doesn't contain anything which isn't a picture of Bob Marley*

(6) _ What does this bag contain?
_ PICTURES OF BOB MARLEY.
→ *this bag doesn't contain anything which isn't a picture of Bob Marley*

The first contribution of this work is to show that the problem of exhaustive readings of indefinites is much more severe than previously described and affects virtually any theories which (i) assumes that the input of the exhaustification procedure is a proposition or set of worlds, (ii) takes the alternatives to be obtained by wholesale replacement of the focus (the propositional exhaustification analyses). The problem thus affects both existing theories, including Rooth (1985); Schwarzschild (1993), as was already noted early on in Van Rooij and Schulz (2007), but also including the later Fox (2007) innocent exclusion approach (despite claims to the contrary). The argument also immediately rules out many variations on these theories that could thus be considered.

This general result explains why existing solutions to the problem of exhaustive readings of indefinites, such as those proposed by Bonomi and Casalegno (1993) and Schulz and Van Rooij (2006), typically involve positing a form of enriched content. This enriched content could be dynamic updates (Schulz and Van Rooij 2006) or event predicates (Bonomi and Casalegno 1993).

The second contribution of this work is to offer another perspective on the problem of exhaustive readings of indefinites. Taking inspiration from Sudo (2023), this proposal extracts from Schulz and Van Rooij (2006) just the amount of dynamic semantics necessary to solve the problem of association with indefinites. The advantage of the proposal over other appraoches in terms of enriched content is its modularity: the proposal can be built around any standard propositional account of exhaustification, e.g. Schwarzschild (1993) or Fox (2007). I argue that this proposal offers certain conceptual advantages over its predecessor (Schulz and Van Rooij 2006), even when the two theories are matched in their predictions.

I will present these contributions in the following order. Section II presents the problem of exhaustive readings of indefinites against different existing proposals and proceed to derive the general impossibility result. Section III consider and dismisses several possible responses to the challenge. Section IV discusses how the problem of exhaustive readings of indefinites is addressed in the existing literature by moving to enriched content and gives preliminary motivations to move beyond these two accounts. Section V presents my own proposal, inspired by dynamic approaches to exhaustification (Van Rooij and Schulz 2007; Sudo 2023). Section VI concludes.

II. The challenge of exhaustive readings of indefinite

1. Preliminaries

The sentence in (8) and the term answer in (9) do not mean the same as the sentence in (7). They in addition strongly suggest that the bag does not contain anything besides *my diary*, the focus of the sentence.

(7) This bag contains my diary.

(8) This bag only contains MY DIARY.

(9) _ What does this bag contain?
_ MY DIARY.

In a tradition[1] going back to Rooth (1985), this exhaustive reading is obtained by contrasting the prejacent with a set of alternatives. Formally, this procedure can be represented with an operator EXH taking two inputs, a prejacent of type *p* and a set of alternatives (type *pt*) and returning a proposition (type *p*).

(10) EXH(prejacent$_p$, alternatives$_{pt}$)

Although not imposed by the format in (10), it is typically assumed that the prejacent and the alternative denotes propositions or sets of worlds. In other words, *p* represents the type *st*.

For instance, Rooth (1985) assumes proposition to be of type *st*. His exhaustification procedure asserts that the prejacent is true and that, if an alternative is not equivalent it, it is false. Applied (7) and (8), this procedure yields the intuitively correct reading that no other entity than my diary are in the bag.

(11) Exhaustification in Rooth (1985)
a. **prejacent:** $\lambda w.\ \text{contains}_w(\text{my-diary})(\text{this-bag})$
b. **alternatives:** $\{\lambda w.\ \text{contains}_w(x)(\text{this-bag}) \mid x \in D_e\}$
c. **Rooth's exhaustification procedure:**
$\text{EXH}(\text{prejacent}, \text{alts}) = \lambda w.\ \text{prejacent}_w \wedge \forall q \in \text{alts}, \rightarrow q \neq \text{prejacent} \rightarrow q_w = 0$

I will call approaches that assume that the prejacent to be a proposition ($p = st$) *propositional exhaustification approaches*. These approaches will later be contrasted with *enriched exhaustification approaches*, which take the input to the prejacent to be more than mere propositions (event predicates, dynamic updates, etc).

In a series of works, Schulz and van Rooij (van Rooij and Schulz 2004; Schulz and Van Rooij 2006; Van Rooij and Schulz 2007) present a number of challenges to existing approaches at the time of their writing. Of interest to us is the problem of accounting for exhaustive readings of indefinites[2], illustrated in (12).

(12) **Context:** *Valuable medieval manuscripts about medicinal herbs were stolen from the library. Maya is the only witness on the scene during midnight and 1am. I report to the police:*
a. Maya only saw A MATH STUDENT.
b. _ Who did Maya see?
_ A MATH STUDENT.
→ *Maya didn't see anyone who wasn't a math student.*

The goal of this section is to show that exhaustive readings of indefinites raise a challenge that applies to new approaches that came after this work but more generally

1 A second tradition, the *structured meaning* approach, represented i.a. by Krifka (1992); Krifka (1993), adopts the position the focus itself is the input to the exhaustification procedure, along with the background, which is the function obtained by abstracting over the focus. My focus for the rest of this work will be on Roothian accounts.

2 Exhaustification of indefinites is also discussed in Krifka (1993), although it is not discussed there as a challenge to any particular approach. The scope solution he invokes is the same as the one discussed critically in III.2.

that the challenge cannot be solved by *any* propositional approach, existing or new. Before stating this general result, I will illustrate the problem with existing problems.

2. Exhaustive readings of indefinites

Let's assume that that the indefinite *a math student* in (12)a and (12)b has the same alternatives as *my diary* in (8) and (9) (more on that below). The challenge for Rooth (1985) is that it predicts contradictory truth-conditions: none of the alternatives are equivalent to the prejacent and so all must be false, i.e. I saw no one. But this contradicts the prejacent *Maya saw a math student.*

(13) Rooth (1985)
  a. **Prejacent:** Maya saw a math student
  b. **Alternatives:** Maya saw X (where X is a plurality)
  c. **Predicted truth-conditions:** $\bot$

In response to a different set of data, Schwarzschild (1993) proposed an improved version of the Roothian exhaustification procedure. His exhaustification procedure asserts that the prejacent is true and that all alternatives *not entailed* by the prejacent are false. But, as pointed out by van Rooij and Schulz (2004), (13)a does not entail *I saw X* for any plurality X and thus Schwarzschild (1993) similarly predicts that all alternatives must be false and the exhaustive reading is a contradiction.

3. Fox (2007) on exhaustive readings of indefinites

Fox (2007)'s influential *innocent exclusion exhaustification* procedure belongs to the class of propositional exhaustification approaches. It is an interesting example to illustrate the challenge of exhaustive readings of indefinites in two respects. First, this approach remains extremely influential in the literature on scalar implicatures up to the present day. Second, Fox (2007) explicitly claims that *innocent exclusion exhaustification* can derive the exhaustive readings of indefinites. On the contrary, I argue here that Fox (2007) does not in fact address the issue.

The innocent exclusion procedure is described below in (14). At a high level, innocent exclusion exhaustification aims to negate as many alternatives as possible, while both maintaining consistency and not discriminating between alternatives. More formally, it constructs maximal sets of alternatives which can be negated consistently with the prejacent (negate as many as alternatives as possible). When there are several such sets, IE exhaustification only negates those alternatives which belong to all maximal sets (no discrimination between alternatives).

(14) Innocent Exclusion Exhaustification
  a. For any set of propositions S,

$$\text{IE}(S) = \bigcap \left\{ \text{alts}' \subseteq \text{alts} \;\middle|\; \begin{array}{l} \text{alts}' \text{ is a maximal subset of alts s.t.} \\ [p \wedge \forall \text{alt} \in \text{alts}', \neg \text{alt}] \text{ is consistent} \end{array} \right\}$$

  b. $\text{EXH}(p, \text{alts}) = p \wedge \forall q \in \text{IE}(p, \text{alts}), \neg q$

Because is is designed to avoid contradictions, IE exhaustification gives hope that it will avoid the problem faced by Rooth and Schwarzschild's theories, which both generated contradictions. To test this, consider (15), repeated from (12).

(15) **Context:** *Valuable medieval manuscripts about medicinal herbs were stolen from the library. Maya is the only witness on the scene during midnight and 1am. I report to the police:*
  a. Maya only saw A MATH STUDENT.

b. _ Who did Maya see?
_ A MATH STUDENT.
→ *Maya didn't see anyone who wasn't a math student.*

For sentences of this kind, we assume that the set of alternatives is as in (16)b, following Fox (2007). Fox (2007) further assumes that the restrictor of the indefinite *math student* has a known and fixed extension. Following him, let us take the set of math students to contain only two people: Lea and Anna. This proves critical, as we'll see below.

Informally, the innocent exclusion exhaustification procedure will run as follows: since I saw a philosophy student, at least one of the two propositions "*Maya saw Lea*" and "*Maya saw Anna*" has to hold. Since these propositions are symmetric, neither one can be excluded by IE exhaustification. Alternatives of the form "Maya saw *x*", where *x* is not a math student, are on the other hand all excludable without contradiction and without breaking symmetry. Negating these alternatives, the resulting reading asserts that Maya saw a math student, i.e. one of Anna and Lea, and that she didn't see any students who didn't study math. This is equivalent to the desired reading in this context.

(16) Fox (2007)
a. **Prejacent:** $\lambda w.\ \exists x, x \in \{\text{Anna}, \text{Lea}\} \wedge \text{saw}_w(x)(\text{Maya})$
b. **Alternatives:** for all individuals *x*, $\lambda w.\ \text{saw}_w(x)(\text{Maya})$
c. **Maximal sets:**
- $\{y \neq \text{Lea} \mid \lambda w.\ \text{saw}_w(y)(\text{Maya})\}$
- $\{y \neq \text{Anna} \mid \lambda w.\ \text{saw}_w(y)(\text{Maya})\}$

d. **Innocently excludable alternatives:**
$\{y \notin \{\text{Anna}, \text{Lea}\} \mid \lambda w.\ \text{saw}_w(y)(\text{Maya})\}$
e. **Exhaustive reading:**
$\lambda w.\ \exists x, x \in \{\text{Anna}, \text{Lea}\} \wedge \text{saw}_w(x)(\text{Maya}) \ \wedge\ \forall y, y \notin \{\text{Anna}, \text{Lea}\} \rightarrow \text{saw}_w(y)(\text{Maya})$
= *Maya saw Anna or Lea but no one who wasn't one of the two*

Problematically, Fox (2007)'s derivation only goes through when the domain for *math students* cannot vary across possible worlds. To illustrate the problem in a drastic light first, consider a case[3] where any set of individuals may be the extension of *math students* in some world of the common ground. In other words, it's not common ground who any of the math students are. Let's further assume that Anna, Lea and Liu are the only relevant people in the domain. In that case, the IE exhaustification procedure fails to negate any alternative. A simple reasoning can help us understand why: in so under-specific a context, there are worlds in which Anna is a math student and Maya only saw her ; there are worlds in which Lea is a math student and Maya only saw her, etc. The existence of these worlds makes the prejacent consistent with the negation of any proper subset of the alternatives. All the alternatives are symmetrical and so none of them can be negated. Formal details are given in (17).

(17) Assumption: for any *S*, there is a world *w* in the common ground such that $[\![\text{maths student}]\!]^w = S$
a. **Prejacent:** $\lambda w.\ \exists x \in [\![\text{maths students}]\!]^w$, Maya $\text{saw}_w\ x$
b. **Alternatives:** for all individuals *x*, $\lambda w.$ Maya $\text{saw}_w\ x$
c. **Maximal sets:**

3 Magri (2009) explicitly claims that the covert exhaustification operator EXH is insensitive to contextual information. If that is so and this is true of *only* too, this assumption would actually not be so extreme but the normal case.

- $\{y \neq \text{Lea} \mid \lambda w.\ \text{Maya saw}_w\ y\}$
- $\{y \neq \text{Anna} \mid \lambda w.\ \text{Maya saw}_w\ y\}$
- $\{y \neq \text{Liu} \mid \lambda w.\ \text{Maya saw}_w\ y\}$

d. **Innocently excludable alternatives:** $\emptyset$

This is an incorrect prediction: exhaustive readings are accessed, even in the absence of any particular knowledge about who the math students are.

This is a limiting case. In the more general case, there are three subsets of individuals: those that we know to be math students (A), those we know not to be math students (B), those for which there is uncertainty (C). The exhaustive reading predicted by Fox (2007) asserts that I saw a math student and that I didn't see any one in B, i.e.:

(18) Maya saw a math student and no one who we know isn't a math student.

This seems incorrect. The sentence's meaning simply does not seem to imply anything about our shared knowledge of who the math students are. I can well report Maya's testimony to the investigators as in (15)a without worrying about who we can identify as a math student and who we cannot.

In conclusion, Fox (2007) does not in fact resolve the challenge of exhaustive readings that Van Rooij and Schulz (2007) leveled at predecessors. This is for a principled reason, as we'll see in the next section.

4. General result

I have reviewed propositional accounts of exhaustive readings and showed that they fail to derive exhaustive readings of indefinites. In this section, I argue that there is a principled reason why these accounts do not succeed: there isn't enough information in the input to the exhaustification procedure (the prejacent and the alternatives) to deliver the right result, if these inputs are propositions ; the same propositions yield different outcomes. If correct, these results make a strong case against propositional exhaustification theories, existing and to be invented, and leads us to rethink what the input to exhaustification is..

Written abstractly, the challenge of exhaustive readings of indefinites is to find a set of alternatives and an exhaustification procedure that will turn (19)a into (19)b.

(19)

a. **Prejacent:**
$\lambda w.\ \exists X \in P_w,\ Q_w(X)$
→ *Maya saw a math student*

b. **Desired result:**
$\lambda w.\ \exists X, P_w(X) \wedge Q_w(X)\ \wedge\ \forall X, \neg P_w(X)\ \rightarrow\ \neg Q_w(X)$
→ *Maya saw a math student and Maya didn't see anything or anyone that wasn't a math student*

The net effect of the exhaustification procedure is to add a conjunct asserting that Maya didn't see any non-math students. To be able to construct (19)b from (19)a and its alternatives, the exhaustification procedure must, loosely speaking, be able to reconstruct the set of math students from the input provided, i.e. the prejacent and the set of alternatives. However, for many reasonable sets of alternatives, the set of math students cannot be reconstructed from the input. This includes all of the set of alternatives in (20), which are inspired by Rooth's theory of alternatives (Rooth 1999); they consist in all objects in a specific type domain.

(20) Different sets of alternatives

a. Individuals as alternatives:

$A_e := \{\lambda w.\, Q_w(Y) \mid Y\}$

→ {I saw Lea, I saw Karl, I saw Anna, I saw Anna and Lea, ...}

b. Individual concepts as alternatives:

$A_{se} := \{\lambda w.\, Q_w(\iota_w) \mid \iota \in D_{se}\}$

→ {I saw Lea, I saw the tallest student, I saw the neighbour, ...}

c. Upward-monotone quantifiers as alternatives:

$A_{(et)t} := \{\lambda w.\, \mathbb{Q}(Q_w) \mid \mathbb{Q} \in D_{(et)t}, \mathbb{Q} \text{ upward monotone}\}$

→ {I saw Lea, I saw two math students, ...}

Formally, the claim is as follows:

(21) **Claim:** There is no operator EXH of type $(st)((st)t)st$ such that for any predicate P and Q:

$\text{EXH}\left(\lambda w.\exists x \in P_w, Q_w(x), A_.\right) = \lambda w.\, \exists x \in P_w,\ Q_w(x) \wedge \forall x \notin P_w, \neg Q_w(x)$

where $A_.$ is one of A_e, A_{se} and $A_{(et)t}$

To show this, it suffices to exhibit two restrictors P and P' such that the sentence "Maya saw a P" (in logical form: $\lambda w.\, \exists X, P_w(X) \wedge Q_w(X))$ and "Maya saw a P'" (in logical form: $\lambda w.\, \exists X, P_w'(X) \wedge Q_w(X))$ are equivalent, but whose exhaustified readings are not. The crucial observation is that, in a Roothian tradition where alternatives are obtained by replacing the focus by items among a certain class, the set of alternatives of the two sentences only depends on Q (i.e. the scope of the indefinite) and is therefore the same for both sentences. Since the inputs to the exhaustification procedure (the prejacent and the alternatives) are equivalent, EXH should, by compositionality, deliver the same proposition.

We start with an example of this which, though flawed, is more intuitive. Let's assume the background information in (22). This information makes (23)a and (23)b perfectly equivalent: detecting rockets entails detecting spaceships, and because of the law of physics assumed in (22), detecting spaceships means that we detect every object in the lowest orbit, in particular rockets.

(22) **Context:** *it is a fact of physics that if the radar detects an object A, it will also detect every object that is bigger than A. Cruisers are the largest ships. There are some cruisers in the radar's range, that's for sure, but there may be other ships, there may be submarines, etc. Let's see what it detects.*

(23) Prejacents

a. The radar detected cruisers.

b. The radar detected ships.

The alternatives obtained by replacing the restrictors in (23) in this example, regardless of how we construe the replacement procedure are exactly the same:

(24) Possible sets of alternatives

a. {the radar detected $x \mid x$ }

b. {the radar detected some $P \mid P$}

c. {the radar detected $Q \mid Q$}

Now, it is clear that the exhaustive readings of the two prejacents, in (25) and (26), is different. While (25)a and (26)a imply (in particular) that no non-rocket spaceships has been detected, (25)b and (25)b carry no such entailment.

(25) *Only*

a. The radar only detected CRUISERS.

b. The radar only detected SHIPS.

(26) Term answers

a. _ What did the radar detect?

_ CRUISERS.

b. _ What did the radar detect?

_ SHIPS.

Because the prejacents are equivalent in both cases and because the alternatives are the same, it is impossible for a propositional exhaustification procedure to deliver a different result for the two cases in a and b.

This in a nutshell is the argument in favor of the claim in (21). But it has a weakness, as hinted earlier: the equivalence of the prejacents is contextual equivalence, obtained by the facts listed in the elaborate context of (22). It has been argued (Magri 2009; Magri 2011) that the exhaustification procedure should be blind to contextually provided information of this sort. In sum, (23)a isn't picking out the same set of worlds as (23)b even though they pick the same subset of world within any common ground that make (22) true. The exhaustification procedure could be sensitive in some way to that difference. However, it turns out that we can construct minimal pairs which are logically equivalent as well. Expressing these logical minimal pairs in a formal language is easy enough, but they are a bit cumbersome in English. For this reason, these examples are relegated to appendix VII.

To summarize, the existence of the minimal pairs in (22) suggest that the propositional content of the prejacent and the alternatives don't contain enough information to deliver the truth-conditions of the exhaustive readings. This is only preliminary: we will explore several ways this conclusion could be resisted in section III and I will argue that these tactics are unsuccessful.

Before we turn to these objections, let me mention a precursor to the challenge raised here, discovered by Groenendijk and Stokhof (1990) and dubbed the "*non-functionality challenge*" by Bonomi and Casalegno (1993). Groenendijk and Stokhof (1990) exhibit the surprising contrast between (27)a and (27)b. While "*a girl walks*" and "*at least one girl walks*" are equivalent, they do not give rise to the same exhaustive reading.

(27) Who walks?

a. A girl.

→ *one and no more than one girl walks and no one who isn't a girl walks.*

b. At least one girl.

→ *one or more girls walk and no one who isn't a girl walks.*

Our challenge is a generalization of the one presented by Groenendijk and Stokhof (1990). Groenendijk and Stokhof (1990) do not explicitly point out the class of theories defeated by their examples as we do here. They note the intuitive paradox and are immediately in a position to resolve it, because they have adopted a structured approach to focus (cf fn. 1), hence are not in the propositional camp. Also added here is a more minimal contrast between pairs of equal complexity.

III. Responses

The previous section presented the challenge in its most basic form. This section defuses a number of responses to the challenges, in light of previous theories.

1. Syntactic alternatives and Katzirian alternatives

So far, the reasoning focused on alternatives which were all the alternatives obtained by replacing the focus ("*a math student*") by all expressions of belonging to a certain semantic type.

(28) Roothian alternatives
a. $A_e = \left\{\lambda w.\, Q_w(Y) \;\middle|\; Y\right\}$
→ {I saw Lea, I saw Karl, I saw Anna, I saw Anna and Lea, ...}
b. $A_{se} = \left\{\lambda w.\, Q_w(\iota_w) \;\middle|\; \iota \in D_{se}\right\}$
→ {I saw Lea, I saw the tallest student, I saw the neighbour, ...}
c. $A_{(et)t} = \left\{\lambda w.\, \mathcal{Q}(Q_w) \;\middle|\; \mathcal{Q} \in D_{(et)t}, \mathcal{Q} \text{ upward monotone}\right\}$
→ {I saw Lea, I saw two math students, ...}

These assumptions about alternatives follow Rooth (1985) closely. It is also quite natural to assume that in the case of question-answer pairs, as in (29), the set of alternatives is obtained from the Hamblin denotation of the question, e.g. "*Who did Maya see?*".

(29) _ Who did Maya see?
_ SOME MATH STUDENTS.

One of the problems of the exhaustive readings of indefinite, as we saw, is that these Roothian alternatives erase all information about *P*, the restrictor of the indefinite. Katzir (2007) and later Fox and Katzir (2011) offer a different perspective on alternatives, which may help here. *Contra* Rooth (1985), Fox and Katzir (2011) take alternatives to be syntactic objects obtained from the prejacent through various replacements. More precisely, the set of alternatives is the set of syntactic objects at most as complex as the prejacent, where syntactic complexity, based on Katzir (2007), is defined as follows:

(30) $S' \prec_C S$ if S' can be derived from S by successive replacements of sub-constituents of S with elements of the substitution source for S in C, $\mathrm{SS}(S, C)$.

(31) $\mathrm{SS}(X, C)$, the substitution source for X in context C, is the union of the following sets:
a. The lexicon
b. The sub-constituents of X
c. Constituents mentioned in C

Since the set of alternatives is defined in part through replacements from the prejacent, the set of alternatives depends on the form of the prejacent. Take (29) as an example. Some alternatives that could be generated from it through the definitions in (30) and (31) are given in (32).

(32) Katzirian alternatives of (29):
{*Maya saw some math students*, *Maya saw some physics students, Maya saw Gilbert, Maya saw some teachers, ...*}

In the account by Fox and Katzir (2011), unlike the Roothian alternatives considered up to this point, the set of alternatives does depend on the restrictor in some fashion. But whether this is sufficient to resolve the problem of exhaustive readings of indefinites is not clear at all.

First, the alternatives of the form *"Maya saw some* MAJOR *students"* where MAJOR ranges over different topics (e.g. math, physics, etc) are problematic. In a situation where students only major in one subject, negating these alternatives might deliver a reading that is superficially equivalent to the one intended, i.e. "*Maya saw some math student but didn't see anyone who wasn't a math student*".

(33) Maya saw some math students
and Maya didn't see any physics students
and Maya didn't see any chemistry students
...

Problematically, the reading in (33) is also be generated in a context where students may double major. However, (33) is clearly incorrect in that case. For instance, if Maya saw a student with the gown traditionally donned by math majours, which double majours are also entitled to wear, and no one beyond that particular student, it isn't false to say:

(34) Maya only saw A MATH STUDENT.

Nor would the following reply be inadequate:

(35) _ Who did Maya see?
_ A MATH STUDENT.

The second reason complexity-based alternatives don't offer a straightforward solution is that our minimal pairs from section III.4 don't seem to differ in complexity:

(36) Only
a. The radar only detected CRUISERS.
b. The radar only detected SHIPS.

(37) Term answers
a. _ What did the radar detect?
_ CRUISERS.
b. _ What did the radar detect?
_ SHIPS.

Finally and more fundamentally, in general, it's unclear what truth conditions are predicted by this account for exhaustive readings of *only* and *term answers* when associated with full DPs. The question is raised even before considering exhaustive readings of indefinites; the challenge starts with examples like (38), as noted by Buccola, Križ, and Chemla (2021).

(38) Exhaustive readings of proper names.
a. Maya only saw OLIVER.
b. _ Who did Maya see?
_ OLIVER

The intuitive truth conditions of this sentence are clear: among the relevant set of people, Maya saw Oliver and no other person. But the predictions of the syntactic approach to alternatives seem *prima facie* incorrect. Since proper names like *Oliver* have little syntactic structure to them, the set of alternatives must include sentences where *Oliver* is replaced by alternatives of similarly low complexity, for instance proper names.

(39) Set of alternatives: {*Maya saw Oliver*, *Maya saw Jade*, *Maya saw Jack*, …}

If (39) is indeed the set of alternatives, the truth conditions would informally translate to: *Maya saw Oliver and no one other than Oliver who can be named by a proper name.* This is clearly not a possible reading of the sentence. There are several ways to address this issue by departing somewhat from Fox (2007) and Fox and Katzir (2011). First, as suggested by REDACTED (p.c.), one could posit that pronouns are always low complexity items that can always be substituted to any DP. The alternatives would then be as in (40). If all individuals can be referred to with an appropriately indexed pronoun, then the expected exhaustive reading "*Maya saw Oliver and no one other than Oliver"* can arise.

(40) Set of alternatives: {*Maya saw Oliver*, *Maya saw him*$_i$, *Maya saw her*$_j$, …}

Second, following Buccola, Kriz, and Chemla (2021), alternatives might be taken to include more conceptual alternatives, i.e. alternatives that are not expressible in English but "in the mind" of every speaker. In this view, even if we don't have a name in English for all individuals in the domain, there is a concept for every individual in the relevant domain.

Third, the set of alternatives could also include answers from the salient question under discussion, be it explicit as in (38)b or implicit as in (38)a. In (38), this question is "*who did Maya see?"* and the answers are presumably propositions of the form "*Maya saw x"* for all *x* in the domain of quantification of *who*.

What these three solutions have in common is that, through some means or other, they make the set of Katzirian alternatives to contain at least the full set of all Roothian alternatives $\{\text{Maya saw } x \mid x \in D_e\}$. Turning back to indefinites, there is no reason why indefinites, which are more complex than the proper names just discussed, shouldn't in the same context have at least the same set of alternatives. With this, we are not too far off from the Roothian alternatives from which we illustrated the puzzle. And so it is unclear that we can evade the conclusions drawn from it.

(41) Maya only saw A MATH STUDENT.

(42) _ Who did Maya see?
_ A MATH STUDENT.

2. Scope

As seen in section II.3 with Fox (2007)'s theory, the problem of exhaustive readings of indefinite only arises when the restrictor of the indefinite varies across worlds.

(43) _ Who did Maya see?
_ SOME MATH STUDENTS

A possible line of defense would be to ensure that, in (43), the restrictor of the indefinite is somehow kept constant. A first attempt could assume that the indefinite *some math students* and other indefinite take scope over the element responsible for the exhaustification procedure. Such an approach to exhaustive readings of indefinites is suggested in e.g. Krifka (1993) and Beaver and Clark (2009). (44)a and (44)b illustrate a possible implementation: the indefinite is raised above the operator responsible for exhaustification and the trace it leaves serves as the semantic focus for the exhaustification procedure. (Note this approach demands that the exhaustification procedure in term answers be not pragmatic but implemented by an operator EXH, as in the grammatical tradition (Chierchia, Fox, and Spector 2012).)

(44) First attempt: a solution by scope
  a. [some math students] λX. [Maya only saw X_F]
  b. _ Who did you see?
   _ [some math students] λX. [EXH <Maya saw X_F>]

Any reasonable theory of exhaustification will guarantee that the boxed constituents in (44) denote the proposition "*Maya saw X and didn't see anyone who wasn't a part of X*". The resulting truth conditions would be completely adequate: *there are some math students such that Maya saw them and she didn't see anyone who wasn't part of them.* This is logically equivalent to our target paraphrase: *Maya saw some math students and no one who wasn't a math student*.

But there is evidence that the scope of the indefinite is not as in (44). First, we can diagnose the putative scope by inserting another operator in the structure. This is done in (45), with the quantifier *every*. (45) uses the so-called "inverse-link construction" (Sauerland 2005; May and Bale 2006; Charlow 2010; Kobele 2010) where the indefinite DP contains the element *every* itself. By placing *every* inside the focused element, I keep the question used for the term answer in (45)b simple and avoid the complexity of introducing quantifiers into question denotations[4].

(45) Diagnosing scope with *every*
  a. ?My album only contains AN AUTOGRAPH OF EVERY CELEBRITY I MET.
  b. _ What does your album contain?
   _ AN AUTOGRAPH OF EVERY CELEBRITY I MET

The most natural reading of (46) is paraphrased as in (46)a. It is a bit difficult to diagnose the scope of the different elements due to the inverse link construction. However, this reading is clearly distinct from one where *every* scopes above *only*, as in (46)b. The reading in (46)b, required by the scope-based analysis, is contradictory.

(46) Possible readings of the sentences
  a. *only* ≫ *every* ≫ *a:*
   my album contains an autograph of every celebrity and nothing that isn't an autograph of a celebrity I met.
  b. every ≫ a ≫ only:
   # for every celebrity, there is an autograph of them such that my album only contains it

Second, the exhaustive reading is also available for items that resist wide scope interpretation. This is the case of bare plurals: as illustrated in (47), these items cannot take wide scope with respect to negation.

(47) The radar didn't detect cruisers.
  a. not ≫ ∃: *there are no cruisers that the radar detected*
  b. *∃ ≫ not: *there are cruisers that the radar didn't detect*

4 For reasons that I do not understand, the introduction of an quantifier intervening between *only* and the focus constituent results in subtly degraded acceptability judgments. But the term answer seems entirely felicitous.

The examples presented in section II.4, repeated below in (48), contained bare plurals. There is no difference between the exhaustive readings of these bare plurals and those obtained for other types of indefinites, cf (49).

(48) Bare plurals
  a. The radar only detected CRUISERS.
  b. _ What did the radar detect?
    _ CRUISERS.

(49) Indefinites
  a. The radar only detected SOME CRUISERS.
  b. _ What did the radar detect?
    _ SOME CRUISERS.

3. Transparency

Related to the scope response just seen, a final possible response is in terms of transparent interpretations[5]. The exhaustification procedure, as formalized, is an intensional operation: its input is a *proposition* (the prejacent), and a set of alternative propositions. In principle, it is therefore possible to interpret certain predicates transparently with respect to the exhaustification operator. Assuming represented world variables and binders, such a transparent interpretation could be represented as (50)a, for the case of *only*. By contrast, the opaque interpretation would obtain with the indexing in (50)b.

(50) Only
  a. $\text{only}_w\ \lambda w'.\ [\text{Maya saw}_{w'}\ \text{a math student}_w]$
  b. $\text{only}_w\ \lambda w'.\ [\text{Maya saw}_{w'}\ \text{a math student}_{w'}]$

The distinction between opaque and transparent interpretations is also sensible for term answers but we need to assume that the exhaustification procedure is performed by a syntactically represented operator EXH.

(51) Term answers
  a. $\text{EXH}_w\ \lambda w'.\ [\text{Maya saw}_{w'}\ \text{a math student}_w]$
  b. $\text{EXH}_w\ \lambda w'.\ [\text{Maya saw}_{w'}\ \text{a math student}_{w'}]$

Under the transparent interpretation, the proposition expressed by the prejacent is an existential statement over a constant restrictor set:

(52) **Prejacent:** $\lambda w'.\ \exists x, \text{maths-student}_w(x) \wedge \text{Maya-saw}_{w'}(x)$

This is promising since, as we saw in section II.3, Fox (2007)'s proposal concerning the problem of exhaustive readings of indefinites works in the special case when the restrictor set is constant across worlds. This suggests a general response: exhaustive readings of indefinites are obtained via transparent interpretations of the indefinite's restrictions. This could therefore solve the under-generation problem of Fox (2007).

5 It seems to me that Beaver and Clark (2009) (p. 99, cf. in particular fn. 11) suggests a similar solution to a different challenge involving the exhaustive readings of definites. Although they phrase their solution in terms of presupposition, it seems to me that transparency is doing the most work in what they propose. In particular, their proposal is vulnerable to the same argument by intervention developed below.

But this is just one part of the puzzle: as we saw, when the restriction of the definite has varying extension across worlds, the innocent exclusion analysis predicts a coherent but unavailable reading, informally paraphrasable as (53). This reading, which corresponds to an opaque interpretation of the indefinite, needs to be ruled out.

(53) I saw a math student and no one who we know isn't a math student.

It is unclear what stipulation would guarantee the impossibility of opaque readings. Assuming such a stipulation can somehow be found, this analysis predicts a correlation between exhaustive readings of the indefinites and transparent interpretations. So the question is: are transparency and exhaustivity correlated in this way?

To test whether exhaustivity truly requires transparency, an intervening intensional environment, like *think*, is needed. The idea is schematized in (54): to achieve a constant restriction for the purpose of exhaustification, the indefinite must be read transparently with respect to *only*/EXH, but this will *ipso facto* mean a transparent interpretation of *a math student* with respect to *think*. This is something that can be read off the truth conditions that speaker reported.

(54) λw_0. only/EXH λw_1. ... think λw_2. ... a math student$_{w_0}$

(55) and (56) are cases in point.

(55) Brian only thinks that my bag contains A MAGIC WAND.

(55) and (56) can definitely be uttered in a situation where Brian believes that magic wands exist but it is common knowledge that they don't. With a transparent interpretation, the restrictor *magic wand* would be empty and the sentence would be a contextual contradiction.

(56) _ What does Brian think that my bag contains?
_ A MAGIC WAND.

The argument isn't conclusive yet: with the intervening quantification introduced by *think*, the prejacent and the alternatives no longer have the logical shape for which the problem of the exhaustive readings of the indefinite was established in section II. Indeed, we studied configurations in which the existential quantification of the indefinite had the highest scope in the prejacent but in the current case, *think* outscopes the indefinite, as shown in (57).

(57) **Prejacent:** $\lambda w'. \forall w'' \sim w', \exists x$, magic-wand$_{w''}(x) \wedge$ my-bag-contains$_{w''}(x)$
where $w_1 \sim w_2$ in case, in w_1, Brian deems w_2 possible

But we can show that the problem of exhaustive readings of indefinites also occurs with this type of logical forms. Recall the cruiser example used in II.4: there, we presented two sentences with equivalent truth-conditions, which yielded different results under exhaustification. The same may be done here under an attitude:

(58) **Context:** *it is a fact of physics that if the radar detects an object A, it will also detect every object that is bigger than A. Brian, you and I know this. Brian believes that there are cruisers operated by extra-terrestrial intelligence in the radar range, but we don't.*

(59) Equivalent prejacents.
a. Brian thinks that the radar detected alien cruisers.

b. Brian thinks that the radar detected alien ships.

(60) *only*

a. Brian only thinks that the radar detected ALIEN CRUISERS.
→ *Brian doesn't think it detected alien ships that are not cruisers.*

b. Brian only thinks that the radar detected ALIEN SHIPS.

(61) Term answers

a. What does Brian think the radar detected?
ALIEN CRUISERS.
→ *Brian doesn't think the radar detected alien ships that are not cruisers.*

b. What does Brian think the radar detected?
ALIEN SHIPS.

As was the case without *think*, the exhaustified readings are not equivalent. (61)a conveys that Brian doesn't think that the radar detected alien spacecrafts which are further away, but (61)b carries no such entailment.

To summarize, exhaustive readings don't seem to require a transparent reading of the indefinite's restrictor. In addition, it isn't clear how one could enforce that such readings are the only one possible, as is required by this line of response.

IV. Enriched meanings

The preceding sections have established that exhaustive readings of indefinite raise a general issue for propositional exhaustification accounts. This issue is robust to variations on our assumptions about structure (scope, transparency), the nature of the alternatives (syntactic and semantic) and the exhaustification procedure itself.

This section presents two accounts that do resolve the challenge raised by exhaustive readings of indefinites: Bonomi and Casalegno (1993) and Van Rooij and Schulz (2007). Presenting these analyses in brief, I will argue that the unifying feature of these solutions is recourse to enriched non-propositional meanings for the prejacent: events for Bonomi and Casalegno (1993), and dynamic propositions for Van Rooij and Schulz (2007). I will argue that, while these analyses appear empirically successful, there remain motivations for moving beyond them.

1. Bonomi and Casalegno (1993)

Bonomi and Casalegno (1993) propose a semantics for *only* in which it applies to event predicates, rather than propositions. In their account, (62)a is rendered as (62)b. This paraphrase correctly implies that Maya didn't see anyone who wasn't a math student: if she had seen a non-math student, there would be an event of Maya seeing something which is not part of Maya seeing a math student.

(62)

a. Maya only saw A MATH STUDENT

b. **Bonomi and Casalegno (1993)'s paraphrase:**
There is an event of Maya seeing a math student.
Every event of Maya seeing something/someone is part of an event of Maya of seeing a math student.

More generally, the exhaustification procedure at the heart of Bonomi and Casalegno's approach may be represented as in (63) (respecting the spirit but not the letter of their analysis). The prejacent is an event predicate (type *vt*), as are the alternatives.

(63) $\text{EXH}(\text{prejacent}_{vt}, \text{alts}_{(vt)t})$
$= \lambda e.\ \text{prejacent}(e)\ \wedge\ \forall q \in \text{alts}, \forall e', q(e') \rightarrow e' \prec e$

Unlike the propositional approach, the event-based approach distinguishes between equivalent prejacents like (64)a and (64)b. An event of the radar detecting ships isn't necessarily an event of detecting cruisers. This is so, even if our contextual assumptions guarantee that the existence of the former type of events implies the existence of events of the latter type.

(64)
a. The radar detected cruisers.
b. The radar detected ships.

With a difference in event mereology, the two prejacents yield different results upon exhaustification, as (66) illustrates.

(65)
a. The radar only detected CRUISERS.
b. The radar only detected SHIPS.

(66)
a. (65)a → *every event of the radar detecting something is part of an event of detecting cruisers*.
b. (65)b → *every event of the radar detecting something is part of an event of detecting ships.*

Bonomi and Casalegno (1993)'s account is empirically very successful. But it could be asked whether event semantics is truly the enrichment that the exhaustification procedure is sensitive to. One reason for skepticism is that events live in a restricted domain – the scope of an event existential. An event-based semantics for *only* would require it to occur in that domain as well.

But *only* seems to have a wider distribution than that. For instance, a number of authors assume that, in event semantics, negation out-scopes event closure (Bäuerle 1987; Champollion 2014; de Groote and Winter 2015). Yet, *only* can out-scope negation, as in (67). So, if these authors are correct, the semantics of *only* cannot involve events.

(67) **Context:** *we are playing hide-and-seek*.
I only didn't see ELLEN.

That being said, other theories exist, according to which negation is an event operator (Krifka 1989; Bernard and Champollion 2023). And so, to embrace Bonomi and Casalegno (1993) in full generality, we must also accept the latter theories over the former. The argument is not fatal to Bonomi and Casalegno (1993) but it leaves one wondering whether a more general theory is possible that wouldn't require *only* to be present in domains where events are present.

2. Van Rooij and Schulz (2007)

Van Rooij and Schulz (2007) invoke a different type of enrichment: dynamic semantics. To appreciate their theory and so as to lay the ground for my own proposal, I will lay it out this proposal in more details.

2.a DPL in an intensional setting

First, let me introduce the dynamic semantics I will be using to describe their account and mine later on. It is a direct semantics based on Groenendijk and Stokhof (1991)'s Dynamic Predicate Logic, with an intensional element. I assume a sentence denotes a certain update of type *ggst*, where *g* is the type of assignment functions and *s* the type of worlds. This can be seen as the type of functions mapping input assignments to world-output assignment pairs. I will call a world-assignment pair (w, g) a model in what follows. Given an input assignment *g*, we can therefore say that a sentence *S* is true at a model (w, g'), if *S* updates *g* to g' in *w*, i.e. $[\![S]\!](g)(g')(w)$

(68) is the update for our example sentence. In worlds where I saw a math student, this denotation takes an input assignment and returns all output assignments which contains at index 17 some math student that I saw. In worlds where no such student exists, the function returns no output context, which is dynamic semantics' representation of falsity.

(68) $[\![$Maya saw a math student$_{17}]\!]$
$= \lambda g.\lambda g'.\lambda w.\ \exists x, x \in \text{math-student}_w \wedge g' = g[17 \to x] \wedge \text{saw}_w(x)(\text{Maya})$

2.b Minimal model exhaustification

To present Van Rooij and Schulz (2007)'s account, we must first define a notion of background. Given a certain pattern of accenting, as in (69), we may define the background to be the denotation of the sentence with the meaning of its accented constituents abstracted away, e.g. (69)b.

(69) Maya only saw A MATH STUDENT.
a. Focus: $[\![$a maths student$]\!]$
b. Background: $\lambda w.\lambda x.$ Maya saw$_w$ x

On the basis of this background, Van Rooij and Schulz (2007) define an ordering of models (world-assignment pairs). Their definition

(70) $(w, g) \prec_B (w', g')$
a. w is just like w', i.e. all predicate extensions are the same in w', as they are in w, except that that the background is smaller: $B(w') \subset B(w)$.
b. $g' = g$

In our example, $(w', g) \prec_B (w, g)$ holds when w' is a world just like w, except that Maya saw less entities in it. An important feature of this ordering relation is that two models can only be compared if they have the same assignment component.

With this ordering defined, the minimal exhaustification procedure of van Rooij and Schulz is given in (71). Informally, given an input assignment *g*, the exhaustified sentence is true at any model (w, g') where the prejacent is true and which are minimal with respect to $\prec_B$.

(71) $\text{EXH}(\text{prejacent}_{ggst}, \text{alts}_{(ggst)t})$
$= \lambda g.\lambda g'.\lambda w.\ \text{prejacent}(g)(g')(w) \wedge \neg\exists w', \exists g', \text{prejacent}(g)(g')(w') \wedge (w', g') \prec_B (w, g)$

2.c Application

Take our example sentence in (68). Starting from an empty input assignment function ($g = []$), the sentence will be true of all models $\langle w, [17 \to x]\rangle$ where x is a math student in w that Maya saw in w. Call this set P (Prejacent).

(72) $P = \{\langle w, [17 \to x]\rangle \mid x \text{ is a math student in } w\}$

The smallest pairs $\langle w, [17 \to x]\rangle$ in P with respect to $\prec_B$ are pairs such that there are no models of P with the same output assignment $\langle w', [17 \to x]\rangle$ and w' is just like w, except that Maya saw less people in it. Note that, by imposing that the output assignment is the same (i.e. $[17 \to x]$ when x is a math student), we are only comparing worlds in which this particular individual x could have been introduced, i.e. worlds in which x is a math student that Maya saw. Keeping fixed the fact that Maya saw this individual x (a math student), the worlds in which Maya saw the least people would seem to be worlds in which she saw x and no one else.

To summarize, the sentence will be true in any world in which Maya saw a math student x and she only saw that student x; these truth condition are entirely adequate. The key mechanism in securing these truth conditions is the ability to keep constant the identity of the math student that Maya saw. This ability is afforded by recourse to the enriched meanings of dynamic semantics.

2.d Difficulties

While this accounts seems promising, the definition of ordering is under-specified in one key respect and this raises questions regarding its empirical success. As the reader recalls, $(w, g) \prec_B (w', g')$ just in case w is just like w' except for the extension of the background B (and $g = g'$). But what does it mean for w to be "just like" w' except in one respect? Intuitively, it does not seem possible to alter one fact about the world (e.g. the set of people Maya saw) without concomitantly altering many other facts (e.g. who was on the premises when Maya was in the library, where in the library she was at 1am, etc).

To put this in starker contrast, consider (73). Suppose it is uttered in a context where Maya is playing a card game where players draw exactly three cards at the beginning of their turn.

(73) Maya only drew hearts$_{19}$.

For concreteness, consider a world w in this context where Maya drew two hearts ($h_1 + h_2$) and a diamond and a g' such that $g'(19) = h_1 + h_2$. Plainly, (73) is false in w. But it isn't clear that Van Rooij and Schulz (2007) predicts it false. Any world w' where Maya draws just h_1 and h_2 is a world where she is playing a rather different game than the one she actually is playing. So, intuitively, w' isn't "just like" w. If it isn't, then the model (w, g') may well be minimal with respect to $\prec_B$. The sentence would be incorrectly be predicted true in w.

Of course, the objection just constructed relies on an intuitive understanding of what "just like" means. It may be claimed that, in the relevant sense, w is in fact just like w'. But the objection underscores a strange prediction of Van Rooij and Schulz (2007): they predict that, under some circumstances, (73) may be judged true even if Maya drew a non-heart card, simply because the world of evaluation happens to be too dissimilar to any world where she drew less cards. In other words, they do not validate the truth conditions we assigned to exhaustive readings of indefinites, cf (74).

(74) Maya drew hearts and didn't draw anything that wasn't a heart.

Since all examples provided in this paper and in Van Rooij and Schulz (2007) obey the truth conditions schema exemplified by (74), there does not seem to be evidence for Van Rooij and Schulz (2007)'s truth conditions. By contrast, the account to be presented will deliver exactly the truth conditions in (74).

V. Proposal

In this last section, I will develop an account of exhaustive reading of indefinites based on enriched meanings and an exhaustification procedure tailored to them, which I cill call *branch-wise-exhaustification*. This account is inspired by Van Rooij and Schulz (2007)'s dynamic exhaustification and also has clear connections to Sudo (2016); Sudo (2023) (cf section V.4 for discussion). The analysis builds upon a general recipe that turns an exhaustification procedure applying to propositional arguments into one that can apply to dynamic propositions.

1. Branch-wise exhaustification

I assume given a certain exhaustification procedure for propositions EXH. For concreteness in the sequel, this will be Schwarzschild's exhaustification procedure, already discussed in section II.2 and formally given in (75).

(75) $\text{EXH}(p_{st}, \text{alts}_{(st)t}) = \lambda w.\, p_w = 1 \wedge \forall q \in \text{alts}, q_w = 1 \rightarrow (p \Rightarrow q)$

I will now define a dynamic exhaustification procedure EXH_{BW}, the *branch-wise exhaustification procedure*, built around the procedure EXH. At a high level, this operator separates out the various outcomes (or "branches") of the update denoted by the prejacent and applies EXH to each of these outcomes independently.

This procedure takes as first input, a prejacent, i.e. a dynamic proposition of type $ggst$. I will continue to assume that alternatives are simple propositions of type st. The possibility of exploiting dynamic alternatives is discussed in section V.4 in connection to Sudo (2016) and Sudo (2023). In short, I will assume EXH_{BW} to have the following form:

(76) $\text{EXH}_{BW}(\text{prejacent}_{ggst}, \text{alts}_{(st)t})$ type $(ggst)((st)t)ggst$

The fundamental idea is that EXH_{BW} applies EXH to each "branch" of an update. Fundamentally, a dynamic proposition p (of type $ggst$) defines for a given input assignment g and output assignment g' a certain proposition $p(g)(g')$: the set of worlds in which g can be updated to g'. We may informally call this proposition the *branch* from g to g'.

As an illustration, consider the sentence in (77)a. This sentence denotes the update in (77)b. Given this denotation, the branch from the input assignment [] to the output assignment [17 → Jane] is the proposition "*Maya saw Jane and Jane is a math student*", as derived in (77)c.

(77)

a. Maya saw a math student.

b. $\lambda g.\lambda g'.\lambda w.\ \exists x, g' = g[17 \rightarrow x] \wedge \text{math-student}_w(x) \wedge \text{saw}_w(x)(\text{Maya})$

c. Branch from [] to [17 → Jane]:

$\lambda w.\ \exists x, [17 \rightarrow \text{Jane}] = [\,][17 \rightarrow x] \wedge \text{math-student}_w(x) \wedge \text{saw}_w(x)(\text{Maya})$

$= \lambda w.\ \exists x, x = \text{Jane} \wedge \text{math-student}_w(x) \wedge \text{saw}_w(x)(\text{Maya})$

$= \lambda w.\ \text{math-student}_w(\text{Jane}) \wedge \text{saw}_w(\text{Jane})(\text{Maya})$

The term "branch" comes from the fact that, in a certain sense, the update in (77)b is the combination of all branches from an input assignment to an output assignment. In addition, it may be noted that the truth-conditions of a sentence against an input assignment g (such as (77)a) can be obtained as the disjunction of the truth conditions of all branches from g to any output assignment. In our example, "*Maya saw a math student*" is the disjunction of propositions of the form "*Maya saw x and x is a math student*"

Given that a branch from g to g' is a simple proposition, it can be an input to the propositional exhaustification procedure EXH. For instance, assume the alternatives to (77) are propositions of the form "Maya saw x" for all relevant individuals x. Then, applying EXH to the branch from [] to [17 → Jane], would lead to the exhaustive reading in (78).

(78) Maya saw Jane and Jane is a math student and, for any one that isn't Jane, Maya didn't see them.

Generalizing to all output assignments, we obtain the following exhaustive reading for each branch:

(79) Maya saw x and x is a math student and, for any one that isn't x, Maya didn't see them.

Taking the union of these propositions delivers exactly the exhaustive reading of the indefinite we have sought hitherto:

(80) There is an x such that Maya saw x and x is a math student and, for any one that isn't x, Maya didn't see them.

This is the spirit of the account. Formally rendered, the branch-wise exhaustification procedure just described is given in (81). It updates g to g' in world w if the prejacent does update g to g' in world w and, furthermore, the exhaustive procedure applied to the branch from g to g' is true in w.

(81) $\text{EXH}_{BW}(p_{ggst}, \text{alts}_{(st)t})$
$= \lambda g.\lambda g'.\lambda w.\ p(g)(g')(w) \wedge \text{EXH}(p(g)(g'), \text{alts})(w)$

Let's apply EXH_{BW} to our flagship examples repeated in (82).

(82) Flagship cases

a. Maya only saw A MATH STUDENT.

b. _ Who did Maya see?

_ A MATH STUDENT.

I assume the set of alternatives is in (83)b. As announced earlier, the branches of the prejacent (cf (83)c) are either proposition of the form "x is a math student and Maya saw x" (or contradictions, in case g' differs from g in more than just the value assigned to index 17). As per the definition (81), we apply the propositional exhaustificational procedure EXH to each such branch (cf (83)d), yielding the proposition "Maya saw x, a philosophy student, and no one but x". Combining it all together, we get the update in (83)e. Informally expressed, this update turns the input assignment into one that contains at index 17 a math student x, only if x is the only individual that Maya saw.

(83) Derivation of (82)

a. **Prejacent:**
$\lambda g.\lambda g'.\lambda w.\ \exists x, g' = g[17 \rightarrow x] \wedge \text{math-student}_w(x) \wedge \text{saw}_w(x)(\text{Maya})$

b. **Alternatives:** $\{\lambda w.\ \text{saw}_w(x)(\text{Maya}) \mid x \in D_e\}$

c. **Branch** $p(g)(g')$ (for a given g and g'):
$\lambda w.\ \exists x, g' = g[17 \rightarrow x] \wedge \text{math-student}_w(x) \wedge \text{saw}_w(x)(\text{Maya})$
$= \lambda w.\ (\exists x, g' = g[17 \rightarrow x]) \wedge (\text{math-student}_w(g'(17)) \wedge \text{saw}_w(g'(17))(\text{Maya}))$

d. **Exhaustified meaning of the branch relative to the alternatives:**
$\text{EXH}(p(g)(g'), \text{alts})$
$= \lambda w.\ \text{math-student}_w(g'(17)) \wedge \text{saw}_w(g'(17))(\text{Maya}) \wedge \forall x \neq g'(17), \neg\text{saw}(x)(\text{Maya})$

e. **Exhaustive reading:**
$\lambda g.\lambda g'.\lambda w.$
$\exists x, x \in \text{maths-student}_w \wedge g' = g[17 \rightarrow x] \wedge \text{saw}_w(x)(\text{Maya}) \wedge$
$\text{math-student}_w(g'(17)) \wedge \text{saw}_w(g'(17))(\text{Maya}) \wedge \forall x \neq g'(17), \neg\text{saw}(x)(\text{Maya})$

2. Properties and consequences of the analysis

The proposal also accounts for some of the variants of the flagship examples that we discussed in the course of the previous sections.

For instance, let's consider the contrast between the a. and the b. sentences of (84) and (85). As the reader recalls, the proposition expressed in the prejacents in a. and b. are contextually equivalent and their alternatives (modulo the discussion in section III.1) are the same but the exhaustive readings are different.

(84) *Only*

a. The radar only detected CRUISERS$_{34}$.

b. The radar only detected SHIPS$_{34}$.

(85) What did the radar detect?

a. CRUISERTS$_{34}$.

b. SHIPS$_{34}$.

In the current approach, the difference follows from the fact that, while the two types of sentences are truth-conditionally equivalent, they do not have the same "branches". The updates denoted by the prejacent of the a. and the b. sentences are given below:

(86) Updates

a. $\lambda g.\lambda g'.\lambda w.\exists X, g' = g[34 \rightarrow X] \wedge \text{cruisers}_w(X) \wedge \text{detected}_w(X)$

b. $\lambda g.\lambda g'.\lambda w.\exists X, g' = g[34 \rightarrow X] \wedge \text{ships}_w(X) \wedge \text{detected}_w(X)$

The branch from g to $g[34 \rightarrow X]$ for these two updates are different. For (86)a, it represents the proposition expressed by "X are cruisers and the radar detected X". For (86)b, it represents the proposition expressed by "X are ships and the radar detected X".

(87) Branch from g to $g[34 \rightarrow X]$

a. (86)a: $\lambda w.\ \text{cruisers}_w(X) \wedge \text{detected}_w(X)$

b. (86)b: $\lambda w.\ \text{ships}_w(X) \wedge \text{detected}_w(X)$

As a result, the propositional prejacent that is input to the non-dynamic exhaustification procedure EXH isn't the same in both cases. And so, even with the same set of alternatives, the outcome of EXH is different, as detailed in (88).

(88) Outcome of EXH

a. EXH((87)a, $\{\lambda w.\ \text{detected}_w(Y) \mid Y\}$)
$= \lambda w.\ \text{cruisers}_w(X) \wedge \text{detected}_w(X) \wedge \forall Y, \text{detected}_w(Y) \rightarrow Y \prec X$

b. EXH((87)b, $\{\lambda w.\ \text{detected}_w(Y) \mid Y\}$)
$= \lambda w.\ \text{ships}_w(X) \wedge \text{detected}_w(X) \wedge \forall Y, \text{detected}_w(Y) \rightarrow Y \prec X$

Combining together the results from the different branches, we derive the the update corresponding to the exhaustive reading, as in (89).

(89) Outcome of EXH$_{BW}$

a. $\lambda g.\lambda g'.\lambda w.\exists X, g' = g[34 \rightarrow X] \quad \wedge\ \text{cruisers}_w(X) \wedge \text{detected}_w(X) \wedge \forall Y, \text{detected}_w(Y) \rightarrow Y \prec X$

b. $\lambda g.\lambda g'.\lambda w.\exists X, g' = g[34 \rightarrow X] \quad \wedge\ \text{ships}_w(X) \wedge \text{detected}_w(X) \wedge \forall Y, \text{detected}_w(Y) \rightarrow Y \prec X$

3. Intervention by *think*

Another example that our analysis must explain is the case of intervening intensional operators as in (90), discussed in section III.3. As we saw, such cases remain problematic for propositional exhaustification approach. But, since the semantics of intensional operators interferes with discourse referents introduction, there is a worry that it might challenge the dynamic approach as well.

(90) Brian only thinks that my bag contains A MAGIC WAND.

To deal with this example, we must lay out some assumptions about how *think* interacts with discourse referent introduction. As is well-known, discourse referents introduced by indefinites in the scope of *"Brian thinks …"* typically cannot be picked up by pronouns in main contexts, cf. (91)a. However, they can be picked up in subsequent intensional contexts, when the intensional context is a subset of Brian's belief worlds, cf. (91)b.

(91) Brian thinks my bag contains a magic wand$_{12}$.

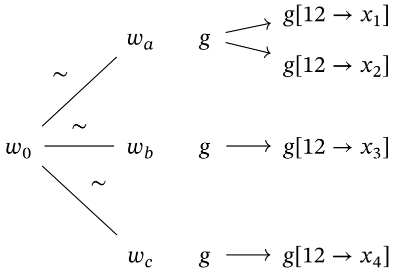
a. # I use it$_{12}$ for nefarious purposes.

b. He suspects I use it$_{12}$ for nefarious purposes.

There are several proposals regarding how to properly derive this behavior. The proposal which I will adopt, captures the common intuition between various approaches to this behavior. To put it informally, *Brian thinks that S* retrieves the individuals introduced by *S* in each of Brian's belief worlds and introduces as discourse referent an individual concept which, in each of Brian's belief worlds, is the individual introduced by *S*. This process is depicted in (92).

(92) Schema for updates

a. Update in individual worlds:



b. Resulting matrix update:

$$g \begin{array}{l} \nearrow \; g\left[12 \rightarrow \iota_1 : \begin{Bmatrix} w_a \rightarrow x_1 \\ w_b \rightarrow x_3 \\ w_c \rightarrow x_4 \end{Bmatrix}\right] \\ \searrow \; g\left[12 \rightarrow \iota_2 : \begin{Bmatrix} w_a \rightarrow x_2 \\ w_b \rightarrow x_3 \\ w_c \rightarrow x_4 \end{Bmatrix}\right] \end{array}$$

Formally, it could[6] be rendered as in (93), which reads: g is updated to g' if and only if for every one of Brian's belief worlds w, S can update g to some g'' and every new discourse referent i introduced by g'' is the individual which, in world w, is picked up by the individual concept $g'(i)$.

(93) $[\![$Brian thinks that $S]\!]$
$\lambda g.\lambda g'.\lambda w.\ \forall w', w' \sim_B w \rightarrow \exists g'', [\![S]\!](g)(g'')(w') \wedge \forall i, i \notin g \rightarrow g'(i)_{w'} = g''(i)$
where (i) $\sim_B$ is Brian's epistemic accessibility relation, (ii) by convention, $h(i) = \#$ and $h(i)_w = \#$ whenever $i \notin h$.

This sketch is not to be taken too literally. What matters for the discussion of exhaustive readings is less the precise nature of the update itself, but the meaning of the different branches. As in the simpler cases, we consider the branch from g to $g' = g[23 \rightarrow \iota]$ for some individual concept ι. While the update in (93) is complex, the branch in (94) simplifies to a simple and legible proposition.

(94) Branch from g to $g'' = g[12 \rightarrow \iota]$
$\lambda w.\ \forall w' \sim_B w, \exists g'', \exists x, g'' = g[12 \rightarrow x] \wedge \text{magic-wand}_{w'}(x) \wedge \text{contains}_{w'}(x) \wedge \forall i, i \notin g \rightarrow g'(i)_{w'} = g''(i)$
$= \lambda w.\ \forall w' \sim_B w, \exists g'', \exists x, g'' = g[12 \rightarrow x] \wedge \text{magic-wand}_{w'}(x) \wedge \text{contains}_{w'}(x) \wedge g'(12)_{w'} = g''(12)$
$= \lambda w.\ \forall w' \sim_B w, \exists x, \text{magic-wand}_{w'}(x) \wedge \text{contains}_{w'}(x) \wedge \iota_{w'} = x$
$= \lambda w.\ \forall w' \sim_B w, \text{magic-wand}_{w'}(\iota_{w'}) \wedge \text{contains}_{w'}(\iota_{w'})$
($\approx$ *Brian thinks that my bag contains ι and that ι is a magic wand*)

With the meaning of the branch established, we now seek to derive the exhaustive reading of such a branch. At this juncture, a problem arises which requires a non-obvious stipulation about the set of alternatives. I will however argue that the problem is an independent one, which is raised by exhaustive readings of indefinites.

To ground intuitions about the problem, consider the exhaustive reading of the sentence in (95), obtained either via *only* or in a term answer, under a De Dicto reading of "the stick I picked up yesterday". The sentence is (95) is similar to one portion of the branch proposition in (94), when ι is the description "the stick I picked up yesterday".

(95) Brian thinks that my bag contains MY DIARY.
a. Brian only thinks that my bag contains MY DIARY.
b. _ What does Brian think that your bag contains?
_ MY DIARY.

The truth conditions of the exhaustive reading are clear when Brian is opinionated about the content of my bag: the sentence asserts that Brian thinks that my bag contains the stick in question and nothing else. The problem is that these truth

6 This entry for *think* will deliver inadequate results in cases where an indefinite in the complement carries an index that already has a value in g (i.e. when a destructive update happens). As it is not really important what the exact formal implementation of (92)b is, I choose to leave the problem open here.

conditions cannot be delivered by negating the simple individual alternatives assumed thus far, as given in (96).

(96) Type *e* alternatives:
Brian thinks my bag contains *x*.

Indeed, assume that Brian has some uncertainty about which stick I picked up yesterday. In this case, none of the alternatives in (96) will be entailed by the prejacent (because there is no particular thing that Brian thinks my bag contains) and all such alternatives will be negatable: *Brian thinks my bag contains the stick I picked up yesterday and, for every x, does not think that my bag contains x.* It is unclear whether this is a possible reading of the sentence, but it certainly doesn't correspond to the truth conditions outlined above.

This new problem bears a family resemblance to the problem of exhaustive readings of indefinites, namely the fact that it occurs when there is uncertainty about individuals identity. However, the problem here can be solved without appeal to enriched meanings but by making adequate changes to the set of alternatives and the exhaustification procedure. By contrast, the problem of exhaustive readings of indefinites, as argued in section II.4, may not be adequately addressed in this way.

Two moves are needed here and I will try my best. The first move is a move to alternatives that contain individual concepts like (97). By incorporating intensionality in the pool of alternatives, we make it possible to get stronger interpretations from exhaustification.

(97) Type *se* alternatives:
Brian thinks my bag contains ι

With the new alternatives, the classical exhaustification procedure however generates contradictions. Indeed, the prejacent (*Brian thinks my bag contains the stick I picked up yesterday*) entails none of the alternatives in (97) when ι is anything but "the stick I picked up yesterday". The exhaustification procedure will therefore negate all of them. This would be contradictory: Brian cannot both think that my bag contains "the stick I picked up yesterday" but no object under any odd description but this one.

To fix this subsidiary issue, we move to an exhaustification procedure that cannot, by design, deliver contradictions. The innocent exclusion procedure of Fox (2007) is one such procedure. Applying it here delivers exactly the right results. First, we determine the maximal sets of alternatives that can consistently be negated with the prejacent. These are the alternatives of the form "*Brian thinks my bag contains* ι" where ι disagrees with s (s representing the individual concept "the stick I picked up yesterday") in at least the world w. The innocently excludable alternatives, which belong to every such set, are those alternatives corresponding to individual concepts that disagree with s in every possible world.

(98) Innocently excludable alternatives
   a. Maximal sets: for any world w, {Brian thinks my bag contains ι | $\iota_w \neq s_w$}
      where $s := [\![\text{the stick I picked up yesterday}]\!]$
   b. Innocently excludable:
      {Brian thinks my bag contains ι | $\forall w, \iota_w \neq \text{stick}_w$}

The proposition obtained by negating these alternatives expresses the desired truth conditions: it asserts that Brian thinks my bag contains the stick I picked up yesterday and, for every individual concept that is necessarily distinct from the stick I picked up yesterday, Brian does not believe that my bag contains it. If Brian is opinionated with respect to the content of my bag, this is the same as saying that Brian doesn't believe there is anything besides the stick in my bag.

With this set of assumptions, the example in (99) may be accounted for.

(99) Brian thinks that my bag contains THE STICK I PICKED UP YESTERDAY.
  a. Brian only thinks that my bag contains THE STICK I PICKED UP YESTERDAY.
  b. _ What does Brian think that your bag contains?
     _ THE STICK I PICKED UP YESTERDAY.

The advantage of the enriched meaning analysis I propose here is that what is adequate for De Dicto definite descriptions in (99) is automatically adequate for exhaustive readings of De Dicto indefinites.

4. The case of dynamic alternatives

I have so far assumed that the alternatives which are input to the exhaustification procedure are mere propositions, rather than dynamic updates. However, this is not very natural, given that the prejacent the alternatives are drawn from is a dynamic update.

In the theory I propose, the dynamic EXH_{BW} is built from a static counterpart EXH. Incorporating dynamic alternatives therefore amounts to finding a way to map the dynamic alternatives onto a set of classical propositions, "*the static alternatives*", which can be serve as input for EXH.

The simplest option to do so is to use a "truth operator" defined below in (100). Given an input assignment, this meta-language operator returns the proposition expressed by the update: following dynamic semantics convention, a sentence is true in w against g if g can be updated to some updates g'. The truth operator represents the passage from the dynamic proposition to this proposition

(100) $\natural u(g) := \lambda w.\exists g', u(g)(g')(w)$

Then, in the definition of branch-wise exhaustification, this operator is used to convert each of the alternatives to a static proposition and the non-dynamic EXH can apply to them, as in (101).

(101) $\text{EXH}_{BW}(\text{prejacent}_{ggst}, \text{alts}_{(ggst)t})$
$= \lambda g.\lambda g'.\lambda w.\ \text{prejacent}(g)(g')(w) \wedge \text{EXH}\left(\text{prejacent}(g)(g'), \left\{\natural\text{alt}(g) \mid \text{alt} \in \text{alts}\right\}\right)$

This option has the advantage of guaranteeing that all the predictions we derived from the branch-wise exhaustification are maintained. Since it effectively erases all dynamic information contained in the alternatives for the purpose of exhaustification, it does not matter whether the alternatives are considered static or dynamic.

There has been a suggestion that the anaphoric potential of alternatives does matter in the exhaustification procedure in Sudo (2023). He proposes an analysis of the multiplicity implicatures of plurals[7], as in (102), that exploits the difference in discourse referents introduced by (102) and (103). These inferences are traditionally

7 Interestingly, other approaches to multiplicity implicatures do rely on other enriched meanings such as events, e.g. Zweig (2008); Ivlieva (2020).

challenging to account for because the natural competitor to (102), (103), is truth-conditionally equivalent to it.

(102) I saw some bikes.
→ *I saw more than one bike.*
(103) I saw some bike.

To solve this, Sudo (2023) proposes (simplifying somewhat) a competition principle that takes into account the referents introduced by both (102) and (103): (102) may not be used in any world where it would introduce exactly the same referents as (103). Assuming (102) introduces a plurality of bikes that I saw, this happens only in worlds where the plurality can only be singularity, i.e. worlds in which I saw just one bike. The inference that the actual world is not a world in which I saw just one bike is precisely the multiplicity inference sought for.

I leave it to open research whether the branch-wise exhaustification approach proposed here can be integrated to Sudo (2023) and how but incorporating the discourse potential of alternatives in the exhaustification procedure seems like a natural continuation of the project here.

VI. Conclusion

This paper has made two contributions. The first has been to revisit the old problem of exhaustive readings of indefinites from Krifka (1993); Van Rooij and Schulz (2007) and show that it presents an insurmountable challenge to approaches that take the input to exhaustification to be mere propositions. The main finding is that there are minimal pairs with equivalent prejacents and, under standard assumption, the same set of alternatives, which yield different exhaustive readings.

The second contribution is to propose an analysis of these readings based on dynamic semantics. This analysis has the conceptual appeal of being modular: it lifts a classic propositional exhaustification procedure into the dynamic realm, allowing it to capture the exhaustive readings of indefinites while retaining its core properties.

Appendix

VII. A logical minimal pair

The generalization upheld in this piece is that the exhaustive reading of the prejacent (104) is (105) when the indefinite is focus-marked.

(104) $\lambda w.\ \exists X, P_w(X) \wedge Q_w(X)$
≈ *Maya saw a math student.*
(105) $\lambda w.\ \exists X, P_w(X) \wedge Q_w(X)\ \wedge\ \forall X, \neg P_w(X) \rightarrow \neg Q_w(X)$
≈ *Maya saw a math student and no one who wasn't a math student.*

I exhibit, in logical form, two logically equivalent prejacents differing only in the restrictor Q for which the exhaustive reading, built on the format of (105), are not equivalent. To do so, I assume given three atomic predicates A, B and C. I construct the scope P and the two restrictors Q and Q' as in (106).

(106)

a. $P_w(x) := A_w(x)$
b. $Q_w(x) := B_w(x)$
c. $Q'_w(x) := B_w(x) \wedge (\exists y, A_w(y) \wedge B_w(y) \wedge C_w(y)) \rightarrow C_w(x)$

To ground intuitions, I also give a cumbersome rendition of this example in plain English:

(107)
a. $P_w(x)$: *Maya saw x*
b. $Q_w(x)$: *x is a math student*
c. $Q'_w(x)$: *x is a math student and if Maya saw a blond math student, then x is blond*

The key element of this construction is that the restrictor Q' is exactly equivalent to Q in all worlds where Maya didn't see a blond math student ($\neg\exists y, A_w(y) \wedge B_w(y) \wedge C_w(y)$), but it is equivalent to "*blond math student*" in worlds where she saw a blond math student. With this observation, it is easy to see why the two prejacents are logically equivalent: this is trivially true in worlds where Maya didn't see a blond math student ; in worlds where did see one, (108)a is true (she saw a math student) and (108)b is too (because she did see a blond math student).

(108) Prejacents
a. $\lambda w.\ \exists X, P_w(X) \wedge Q_w(X)$
b. $\lambda w.\ \exists X, P_w(X) \wedge Q'_w(X)$

But we may also establish it formally:

(109) $\exists X, P_w(X) \wedge Q'_w(X)$
$\Leftrightarrow \exists x, A_w(x) \wedge B_w(x) \wedge (\exists y, A_w(y) \wedge B_w(y) \wedge C_w(y)) \rightarrow C_w(x)$
(by definition)
$\Leftrightarrow \exists x, A_w(x) \wedge B_w(x) \wedge \neg(\exists y, A_w(y) \wedge B_w(y) \wedge C_w(y)) \vee C_w(x)$
(by definition of $\rightarrow$)
$\Leftrightarrow \exists x, (A_w(x) \wedge B_w(x) \wedge \neg(\exists y, A_w(y) \wedge B_w(y) \wedge C_w(y))) \vee \exists x, A_w(x) \wedge B_w(x) \wedge C_w(x)$
(distributivity)
$\Leftrightarrow (\exists x, A_w(x) \wedge B_w(x)) \wedge \neg(\exists y, A_w(y) \wedge B_w(y) \wedge C_w(y)) \vee \exists x, A_w(x) \wedge B_w(x) \wedge C_w(x)$
(removing from scope of first existential, elements that don't depend on x)
$\Leftrightarrow (\exists x, A_w(x) \wedge B_w(x)) \vee \exists x, A_w(x) \wedge B_w(x) \wedge C_w(x)$
($p \vee q$ equivalent to $p \vee (\neg p \wedge q)$)
$\Leftrightarrow \exists x, A_w(x) \wedge B_w(x)$
$\Leftrightarrow \exists x, P_w(x) \wedge Q_w(x)$
(by definition)

The exhaustive readings (constructed following the schema in (105)) are however not equivalent. They will yield the same truth value in any world where Maya didn't see a blond math student but, in worlds where Maya did see a blond student, (110)b will make the stronger claim that she only saw blond math students.

(110) Exhaustive readings
a. $\lambda w.\ \exists X, P_w(X) \wedge Q_w(X)\ \wedge\ \forall X, \neg P_w(X) \rightarrow \neg Q_w(X)$
$\approx$ *Maya saw a math student and no one who wasn't a math student.*
b. $\lambda w.\ \exists X, P_w(X) \wedge Q'_w(X)\ \wedge\ \forall X, \neg P_w(X) \rightarrow \neg Q'_w(X)$
$\approx$ *Maya saw a math student and no one who wasn't a math student who is blond if Maya saw a blond math student.*

To establish non-equivalence formally, we assume a world w with the following extensions for the atomic predicates A, B and C:

(111)

a. $A_w = \{a, b\}$
b. $B_w = \{a, b, c\}$
c. $C_w = \{a\}$

By our definitions, this means the following extensions for the scope and restrictors of the indefinite:

(112)

a. $P_w = \{a, b\}$
b. $Q_w = \{a, b, c\}$
c. $Q'_w = \{a\}$

It is clear that that two prejacents are true: both Q and Q' intersect P. But the exhaustive readings don't have the same truth value: (110)a is true while (110)b is false.

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